2.086 Numerical Computation for Mechanical Engineers

MINI-QUIZ 1 — Fall 2014

You may refer to the textbook, lecture notes, MATLAB® tutorials, and other class materials as well as your own notes and scripts.

You may use a calculator (for simple arithmetic operations and function evaluations). However, laptops, tablets, and smartphones are not permitted.

You have 20 minutes of recitation to complete the mini-quiz.

NAME _____

There are a total of 100 points: four questions, each worth 25 points.

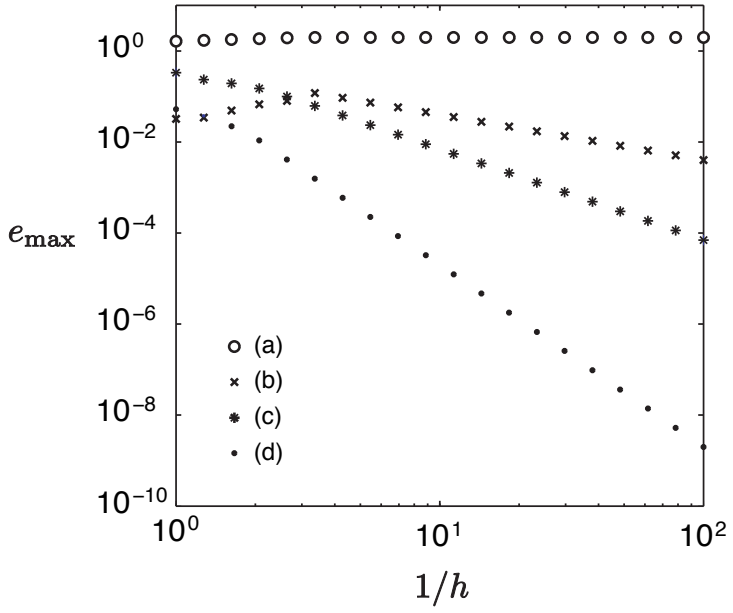
All questions are multiple choice; circle one and only one answer.

We provide two blank pages at the end of the quiz which you may use for any derivations, but note that we will *only grade your multiple choice selections*.

This (same) quiz will be administered in all recitations sections. *You may not discuss this quiz with other people until December 19*.

In this quiz, a *smooth function* denotes a function which has an infinite number of derivatives.

Question 1 (25 points). From the log-log plot below, choose the line that corresponds to a first-order convergence behavior (in the asymptotic regime). Note that $e_{\max}$ is the global interpolation error (associated with the interpolation of some function) and h is the discretization parameter associated with N equispaced points.



Question 2 (25 points). In order to save time when evaluating a particular smooth function $f(x)$ that is unfortunately not one of MATLAB'S built in functions, a student decides to tabulate the values of $f(x)$ at N regularly spaced points and use interpolation to calculate function values at arbitrary values of x (within a specified, fixed range $a \leq x \leq b$). In order to determine how many interpolation points to use, she runs an experiment that shows that with $N = 20$, and using piecewise-linear interpolation, the maximum error in the range $a \leq x \leq b$ is 5%. In order to ensure that the maximum error is less than 1%, the smallest number of interpolation points that can be used (using piecewise-linear interpolation) is

(a) 40

(b) 60

(c) 100

(d) 400

Note that the student assumes, and you may assume, that for $N \geq 20$ we are in the asymptotic regime of the convergence behavior.

Question 3 (25 points). You are given the value of the function $f(x)$ at three points:

x	$f(x)$
0	1.5000
1	4.0774
2	11.0836

The value of the *piecewise-linear interpolant* of $f(x)$ at $x = 1.5$ is

(a) 1.5000

(b) 4.0774

(c) 7.5805

(d) 8.2334

(e) 11.0836

Question 4 (25 points). Consider interpolation of a smooth function $f(x)$ in the range $a \leq x \leq b$ using $N - 1$ equisized segments (N equispaced points). Which of the following statements is <u>not</u> correct?

(a) In the limit $N \to \infty$ the cost (both Offline and Online) of piecewise-constant left-endpoint and piecewise-linear interpolation is similar (of the same order).

(b) Piecewise-linear interpolation represents constant and linear functions exactly.

(c) For N finite and small (e.g. $N = 10$), it is possible—for some functions $f(x)$—that piecewise-constant left-endpoint interpolation will be more accurate than piecewise-linear interpolation for some values of x.

(d) For any function $f(x)$ which is monotonically increasing, the piecewise-linear interpolant of the function will be less than or equal to $f(x)$ for all x, $a \leq x \leq b$.

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