Plate Buckling

ref. Hughes Chapter 12

Buckling of a plate simply supported on loaded edges treated as a wide column results in similar Euler stress, with EI replaced byD*(b):

$$P_e := \frac{\pi^2 \cdot D \cdot b}{a^2}$$

dividing by area (b*t):

$$\sigma_e := \frac{\pi^2 \cdot D}{a^2 \cdot t}$$

$$D := \frac{E \cdot t^3}{12 \cdot \left(1 - \nu^2\right)}$$

eqn 9.1.5, 1/b implied

Buckling of a simply supported plate. i.e. simply supported on all four sides

deflected shape is represented by sine waves in x and y:

$$w(x,y) := \sum_m \sum_n C_{m,n} \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b}\right)$$

m and n are the number of half waves in deflection eqn 12.1.3

$$\sigma_{acr} := \frac{\pi^2 \cdot a^2 \cdot D \cdot \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}{t \cdot m^2}$$

argument in text for minimum stress can specify n = 1, but m not clear

$$\sigma_{acr} := k \cdot \frac{\pi^2 \cdot D}{b^2 \cdot t}$$

with n = 1, can express in form where k is buckling coefficient in equation N.B. shift to plate width b in denominator

eqn 12.1.5

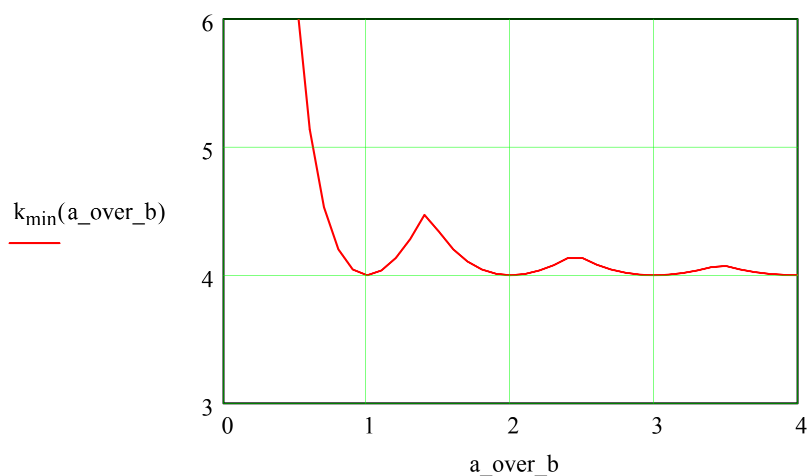
$m := 1 .. 2 \quad n := 1 .. 2$

$a\_over\_b := 0.5, 0.6 .. 4$

$$k(a\_over\_b, m) := \left(\frac{m}{a\_over\_b} + \frac{a\_over\_b}{m}\right)^2$$

eqn 12.1.6

$$k_{min}(a\_over\_b) := \min\left(\begin{pmatrix} k(a\_over\_b, 1) \\ k(a\_over\_b, 2) \\ k(a\_over\_b, 3) \\ k(a\_over\_b, 4) \end{pmatrix}\right)$$



therefore for long plates, simply supported on loaded ends k = 4

$$\sigma_{acr} := 4 \cdot \frac{\pi^2 \cdot D}{b^2 \cdot t}$$

very wide i.e. a/b -> 0, approaches Euler

$$\sigma_{acr}(a\_over\_b) := k_{min}(a\_over\_b) \cdot \frac{\pi^2 \cdot D}{b^2 \cdot t}$$

as a<<b

$$k \rightarrow \left(\frac{b}{a} + \frac{a}{b}\right)^2 = \left(\frac{b^2 + a^2}{a \cdot b}\right)^2 \rightarrow \left(\frac{b}{a}\right)^2$$

a square column meets these boundary conditions hence will buckle with each edge forming half sine wave

deflection in half sine waves approaching square for large a/b; long plate loaded on end simply supported on all sides

Plate loaded on all four sides; σ_{ax} in a direction, σ_{ay} in b direction

Again taking two half wave sine series using energy methods, results in combination expression for both stresses, again taking minimum of straight lines from:

$$\left[\left(\frac{m}{\alpha}\right)^2\cdot\frac{\sigma_{ax}}{\sigma_{axcr1}} + n^2\cdot\frac{\sigma_{ay}}{\sigma_{axcr1}}\right] = \frac{1}{4}\cdot\left[\left(\frac{m}{\alpha}\right)^2 + n^2\right]^2$$

$$\sigma_{acr} := 4\cdot\frac{\pi^2\cdot D}{b^2\cdot t}$$

critical stress with only σ_{ax}

$$\sigma_{axcr1} := \sigma_{acr}$$

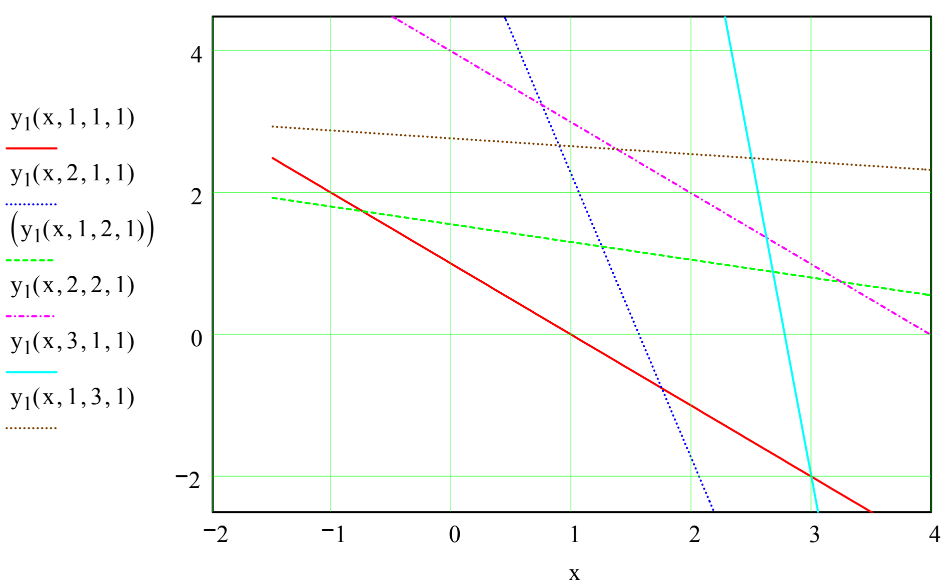
$$y(m,n,\alpha) := \frac{\sigma_{ay}}{\sigma_{axcr1}} \qquad x(m,n,\alpha) := \frac{\sigma_{ax}}{\sigma_{axcr1}} \qquad \alpha := \frac{a}{b}$$

$$y_1(x,m,n,\alpha) := \frac{1}{4\cdot n^2}\cdot\left[\left(\frac{m}{\alpha}\right)^2 + n^2\right]^2 - \left(\frac{m}{n\cdot\alpha}\right)^2\cdot x$$

$x := -1.5, -1.45 .. 4$

fig 9.5 in T&G

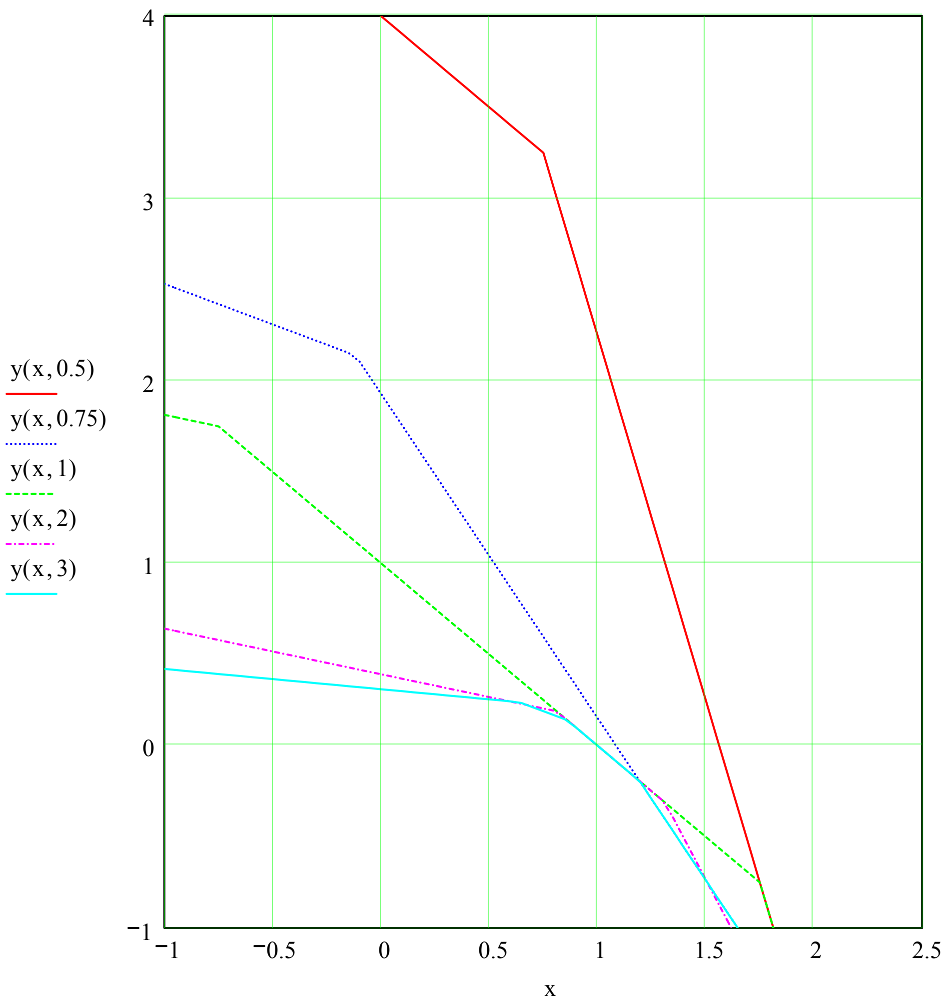
$y_1(x,m,n,\alpha), y_1(x,m,n,\alpha), (y_1(x,m,n,\alpha)), y_1(x,m,n,\alpha), y_1(x,m,n,\alpha), y_1(x,m,n,\alpha)$



Buckling stresses of biaxially loaded simply supported plates only taking m=n=4 terms. Effect of higher terms would be seen in lower right corner asymptote.

$$x := -1, -0.95 .. 2.5$$

$$y(x,\alpha) := \min\left(\begin{pmatrix} y_1(x,1,1,\alpha) & y_1(x,1,2,\alpha) & y_1(x,1,3,\alpha) & y_1(x,1,4,\alpha) \\ y_1(x,2,1,\alpha) & y_1(x,2,2,\alpha) & y_1(x,2,3,\alpha) & y_1(x,2,4,\alpha) \\ y_1(x,3,1,\alpha) & y_1(x,3,2,\alpha) & y_1(x,3,3,\alpha) & y_1(x,3,4,\alpha) \\ y_1(x,4,1,\alpha) & y_1(x,4,2,\alpha) & y_1(x,4,3,\alpha) & y_1(x,4,4,\alpha) \end{pmatrix}\right)$$



Biaxial loading all edges clamped: σ_{ax} is relative to π^2*D/(b²*t)

$$\sigma_{ax}(\alpha, \sigma ay\_over\_\sigma ax) := \frac{4\cdot\left(\frac{3}{\alpha^2} + 3\cdot\alpha^2 + 2\right)}{3\cdot\left(1 + \alpha^2\cdot\sigma ay\_over\_\sigma ax\right)}$$

eqn 12.3.3 in T&G form

$\sigma_{ax}(1.,0) = 10.667$ $\sigma_{ax}(1,-2) = -10.667$

tbl. 12-3
10.65, 20

$\sigma_{ax}(1,1) = 5.333$ $\sigma_{ax}(1,0.25) = 8.533$

5.61, 8.80

per T&G good for shape close to square, data does not compare well with Hughes tbl. 12.3 or T&G tbl 9-15

plate buckling due to pure shear: **simply supported on four sides:** T&G article 9.7, page 379ff

$$\tau_{cr} := \frac{-\pi^2}{32\cdot\beta}\cdot\left(\frac{\pi^2\cdot D}{b^2\cdot h}\right)\cdot\frac{1}{\lambda}$$

$$\sigma_{ref} := \frac{\pi^2\cdot D}{b^2\cdot h}$$

h = thickness = t

from five equations

$\beta := 1, 1.01 .. 2$ β = a/b

$$\tau_{cr}(\beta) := \frac{-\pi^2}{32\cdot\beta}\cdot\left(\frac{\pi^2\cdot D}{b^2\cdot h}\right)\cdot\frac{1}{\lambda(\beta)}$$

$$\lambda(\beta) := \sqrt{\frac{\beta^4}{81\cdot\left(1+\beta^2\right)^4}\cdot\left[1 + \frac{81}{625} + \frac{81}{25}\cdot\left(\frac{1+\beta^2}{1+9\cdot\beta^2}\right)^2 + \frac{81}{25}\cdot\left(\frac{1+\beta^2}{9+\beta^2}\right)^2\right]}$$

k in $\tau_{cr} := k\cdot\frac{\pi^2\cdot D}{b^2\cdot h}$ becomes $k(\beta) := \frac{\pi^2}{32\cdot\beta}\cdot\frac{1}{\lambda(\beta)}$ close for β = <2, no good elsewhere

it's probably ok by T&G, for few equations

compare with simple curve fit $k_1(\beta) := 5.35 + \frac{4}{\beta^2}$

exact solution for very long plates β = a/b -. ∞ is k = 5.35. At β = 1 k = 9.34 exact.

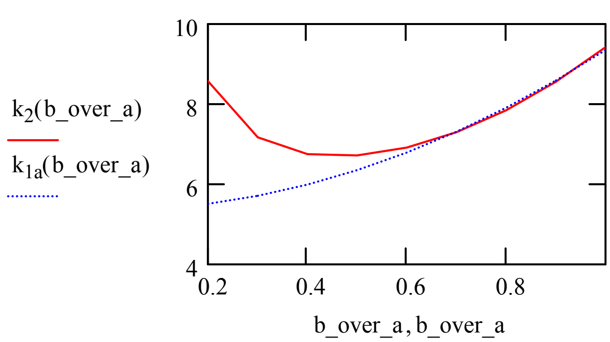
$\frac{k(1)}{9.34} - 1 = 0.0088$ less than 1% difference

more complexity doesn't buy anything

$$k_2(b\_over\_a) := \frac{\pi^2\cdot b\_over\_a}{32}\cdot\frac{1}{\lambda\left(\frac{1}{b\_over\_a}\right)}$$

$$k_{1a}(b\_over\_a) := 5.35 + 4\cdot b\_over\_a^2$$

$b\_over\_a := 0.2, 0.3 .. 1$



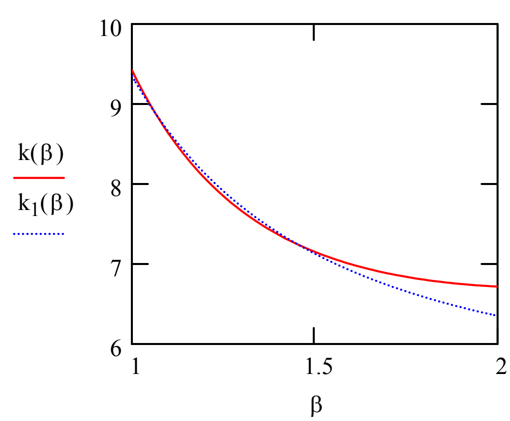
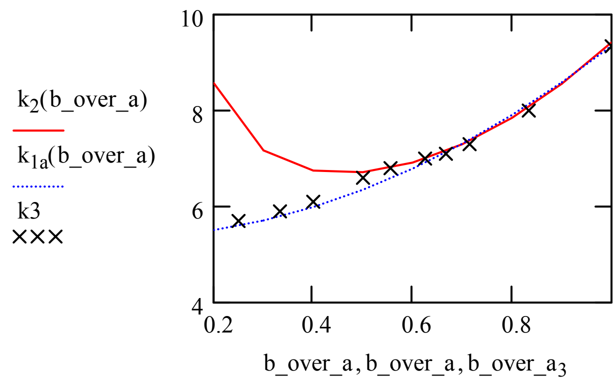


table 9-10 in T&G page 382; values of k in $\tau_{cr} := k \cdot \frac{\pi^2 \cdot D}{b^2 \cdot h}$ from the results of the solution by determinants = 0.

$$tbl\_9\_10 := \begin{pmatrix} 1.0 & 1.2 & 1.4 & 1.5 & 1.6 & 1.8 & 2.0 & 2.5 & 3 & 4 \\ 9.34 & 8.0 & 7.3 & 7.1 & 7.0 & 6.8 & 6.6 & 6.1 & 5.9 & 5.7 \end{pmatrix} \quad i := 0..9$$

$$k3_i := tbl\_9\_10_{1,i} \qquad b\_over\_a3_i := \left(\frac{1}{tbl\_9\_10_{0,i}} \right)$$



clamped on four sides critical shear stress

T & G only provides infinitely long value of $\tau_{cr} := 8.98 \cdot \frac{\pi^2 \cdot D}{b^2 \cdot h}$ and then plots combination

Interaction formula for combination of shear stress and axial load

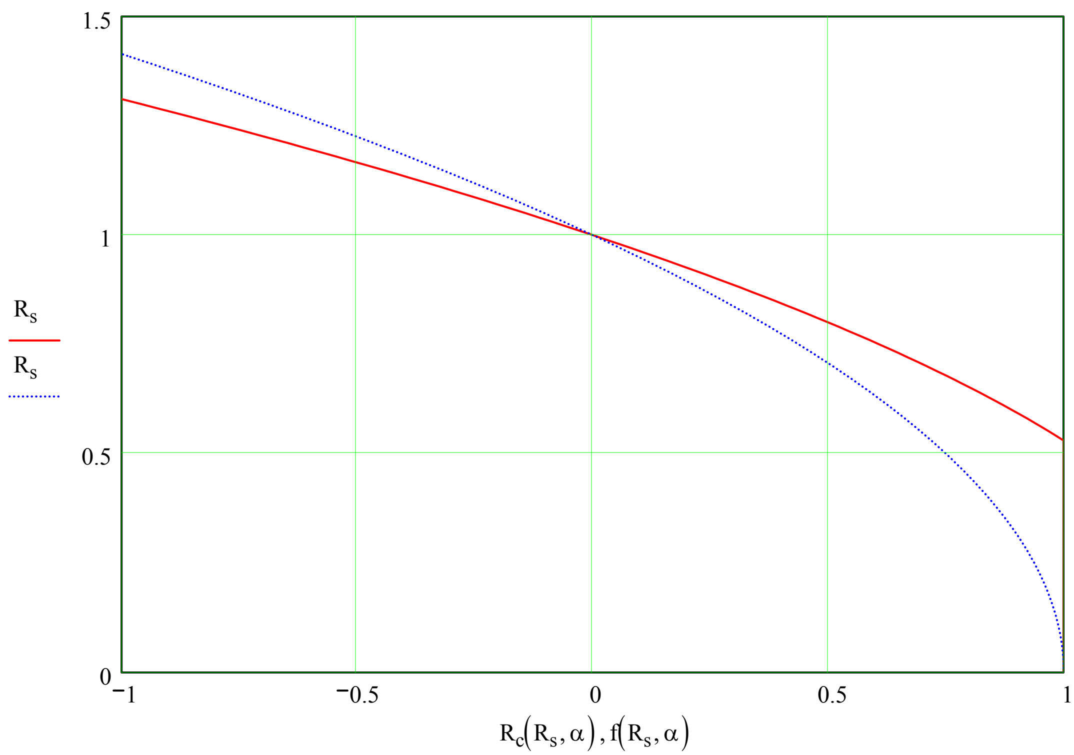
from ref 10 (in AA library) when a/b > 1, problem is one of axial stress, and parabola is good model. When a/b is less than one, it is the equivalent of a transverse stress with shear. Parabola is not a good fit. Hence Hughes model based on a/b.

$$R_c(R_S, \alpha) := \text{if}\left[\alpha > 1, 1 - R_S^2, \text{if}\left[R_S^2 > \frac{3}{8}\cdot(1-\alpha), \frac{1 - R_S^2}{(1 + 0.6\cdot\alpha)}\cdot 1.6, 1\right]\right]$$ eqn 12.4.5

$R_S := 0, 0.01 .. 1.5$ $\qquad$ $f(R_S, \alpha) := 1 - R_S^2$

$\alpha := 0.25$ looks a transverse stress with a/b = 4.

since α <1, parabola doesn't fit well with Rc(Rs,α). Curve lays on top when α>1, (by definition).



Ultimate strength of plates:

$$\xi(\beta) := 1 + \frac{2.75}{\beta^2}$$

$$\beta := \frac{b}{t} \cdot \sqrt{\frac{\sigma_Y}{E}}$$

$$Es\_over\_E(\beta) := 0.25 \cdot \left(2 + \xi(\beta) - \sqrt{\xi(\beta)^2 - \frac{10.4}{\beta^2}}\right)$$ 12.6.4

$$\sigma r\_over\_\sigma y := 0.0$$ for curve in text: set this parameter to 0.1

$$\sigma au\_over\sigma y(\beta) := 0.25 \cdot \left(2 - 4 \cdot \sigma r\_over\_\sigma y + \xi(\beta) - \sqrt{\xi(\beta)^2 - \frac{10.4}{\beta^2}}\right)$$ 12.6.5 with reduction $\sigma r\_over\_\sigma y = 0$

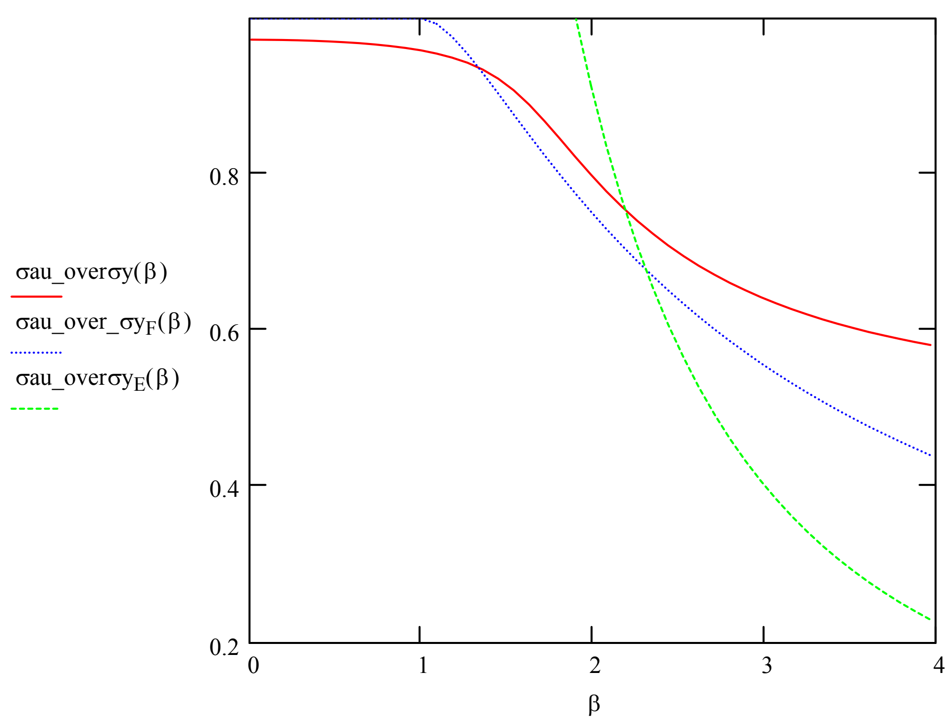
Faulkner

$$\sigma au\_over\_\sigma y_F(\beta) := \text{if}\left(\beta < 1, 1, \frac{2}{\beta} - \frac{1}{\beta^2}\right)$$

$$Ets\_over\_E(\beta) := \text{if}\left[\beta < 1, 0, \text{if}\left[\beta > 2.5, 1, \frac{2 \cdot (\beta - 1)}{\beta}\right]\right]$$ to correct for residual stress

$$\sigma au\_over\sigma y_E(\beta) := \frac{3.62}{\beta^2}$$ for reference

$$\beta := 0.01, 0.1 .. 4$$



similar to fig 12.25 in text

bringing in some other standards: AISI and UK (new) from Professor Wirrzbicki's Manual for Crash Wothiness Engineering Feb 1989 CTS MIT and in 13.019 handout validation of plate buckling

note: be/b is another way of representing peak load

$$b_e \cdot \sigma_Y = b \cdot \sigma_{cr}$$

$$\sigma_{cr\_over\_\sigma Y}(\beta) := \left(\frac{1.9}{\beta}\right)^2$$

$$\sigma_{Y\_over\_\sigma cr}(\beta) := \left(\frac{\beta}{1.9}\right)^2$$

$$C(\beta) := \mathrm{if}\left(\beta > 1.25, \frac{2.25}{\beta} - \frac{1.25}{\beta^2}, 1\right)$$ ABS Alaa

$$\sigma au\_over\sigma y_E(\beta) := \mathrm{if}\left(\beta > 1.9, \frac{3.62}{\beta^2}, 1\right)$$ for reference

$$\beta_1 := 1.5$$

$$be\_over\_b_{uk}(\beta) := \mathrm{if}\left[\beta > \beta_1, \left[1 + 14\cdot\left(\sqrt{\sigma_{Y\_over\_\sigma cr}(\beta)} - 0.35\right)^4\right]^{-0.2}, 1\right]$$

$$be\_over\_b_{aisi}(\beta) := \mathrm{if}\left[\beta > \beta_1, \sqrt{\sigma_{cr\_over\_\sigma Y}(\beta)}\cdot\left(1 - 0.218\cdot\sqrt{\sigma_{cr\_over\_\sigma Y}(\beta)}\right), 1\right]$$

$$\beta := 0.5, 0.6 .. 10$$