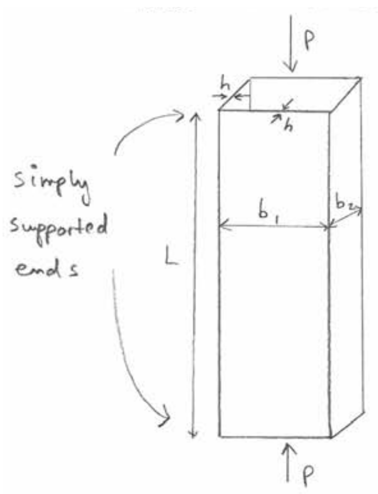
Recitation 9

Buckling of Sections

Consider the box column shown:

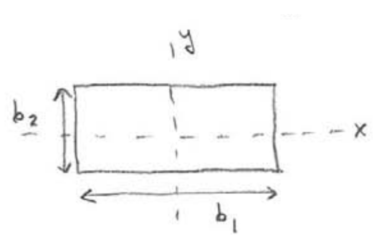


Find the critical buckling load.

Column can buckle
globally (Euler) or
locally (plate)

Global (Euler) Buckling

$$P_c = \frac{\pi^2 EI}{L^2}$$



$$I_x = 2\left(\frac{b_1h^3}{12} + b_1h\left(\frac{b_2}{2}\right)^2\right) + 2\frac{hb_2^3}{12}$$
$$= \frac{b_1h^3}{6} + \frac{b_1b_2^2h}{2} + \frac{b_2^3h}{6}$$
$$I_y = 2\left(\frac{b_2h^3}{12} + b_2h\left(\frac{b_1}{2}\right)^2\right) + 2\frac{hb_1^3}{12}$$
$$= \frac{b_2h^3}{6} + \frac{b_1^2b_2h}{2} + \frac{b_1^3h}{6}$$

$I_x < I_y \rightarrow$ Will buckle about x-axis

So $P_c = \frac{\pi^2 E}{L^2}\left(\frac{b_1h^3}{6} + \frac{b_1b_2^2h}{2} + \frac{b_2^3h}{6}\right)$

Local (Plate) Buckling

Treat each side as an individual plate.

* Assume *uniform compression* → stress in each plate is the same.

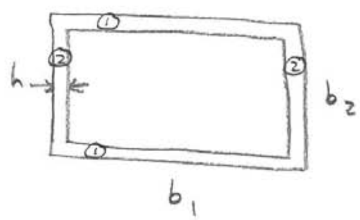


Plate ①: $\sigma_1 = \dfrac{P_1}{hb_1}$

Plate ②: $\sigma_2 = \dfrac{P_2}{hb_2}$

For a plate simply supported on the loaded edges:

$$P_c = \frac{k_c \pi^2 D}{b} \quad \text{(where } k_c \text{ depends on BC on other sides and dimensions)}$$

$$\left(D = \frac{Eh^3}{12(1-\nu^2)}\right)$$

So

$$\sigma_{cr,1} = \frac{P_{c1}}{hb_1} = \frac{k_{c1}\pi^2 D}{hb_1^2}$$

and

$$\sigma_{cr,2} = \frac{P_{c2}}{hb_2} = \frac{k_{c2}\pi^2 D}{hb_2^2}$$

All plates buckle at the same time, so

$$\sigma_{cr,1} = \sigma_{cr,2}$$

$$\frac{k_{c1}\pi^2 D}{hb_1^2} = \frac{k_{c2}\pi^2 D}{hb_2^2} \;\rightarrow\; k_{c2} = k_{c1}\left(\frac{b_2}{b_1}\right)^2$$

$$\text{Total load} = 2P_1 + 2P_2$$

$$\rightarrow\; P_{c,tot} = 2P_{c1} + 2P_{c2} = 2\left(\frac{k_{c1}\pi^2 D}{b_1} + \frac{k_{c2}\pi^2 D}{b_2}\right)$$

$$= 2\pi^2 D k_{c1}\left[\frac{1}{b_1} + \left(\frac{b_2}{b_1}\right)^2 \cdot \frac{1}{b_2}\right]$$

$$= \frac{2\pi^2 D k_{c1}}{b_1}\left(1 + \frac{b_2}{b_1}\right)$$

But what is k_{c1}??
- In general, the adjacent plates on the unloaded edges will cause a bending moment (somewhere between simply supported and fully clamped)
(Plot on page 16 gives k_{c1} as function of $\frac{b_2}{b_1}$)
(assumes $L/b_1 > 5$)

Example

$h = 2$ mm
$b_1 = 100$ mm
$b_2 = 50$ mm

Find the length, L, that marks transition between global and local buckling.

$$\begin{aligned}
P_{\text{c,global}} &= \frac{\pi^2 E}{L^2}\left(\frac{b_1 h^3}{6} + \frac{b_1 b_2^2 h}{2} + \frac{b_2^3 h}{6}\right) \\
&= \frac{\pi^2 E}{L^2}\left(\frac{100(2)^3}{6} + \frac{100(50)^2(2)}{2} + \frac{50^3(2)}{6}\right) = \frac{\pi^2 E}{L^2}(291,800 \text{ mm}^4) \\
P_{\text{c,local}} &= \frac{2\pi^2 D k_{c1}}{b_1}\left(1 + \frac{b_2}{b_1}\right) \quad \text{(From plot: } k_{c1} \simeq 5.2\text{)} \\
&= \frac{2\pi^2 E h^3 k_{c1}}{b_1(12)(1-\nu^2)}\left(1 + \frac{b_2}{b_1}\right) \\
&= \frac{2\pi^2 E(2)^3(5.2)}{100(12)(1-0.3^2)}(1+0.5) = \pi^2 E(0.114 \text{ mm}^2)
\end{aligned}$$

Let $P_{\text{c,global}} = P_{\text{c,local}}$ and solve for L:

$$\begin{aligned}
\frac{\pi^2 E}{L^2}(291,800) &= \pi^2 E(0.114) \\
L \simeq 1600 \text{ mm} &= \boxed{1.6 \text{ m}}
\end{aligned}$$

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