MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF OCEAN ENGINEERING

13.811 Advanced Structural Dynamics and Acoustics

Second Half - Problem Set 1 Solution

Problem 1

(a) Using equation (2.99) of text and substituting $\dot{w}_o$ for surface velocity,

$$\dot{W}(k_x, k_y, 0) = \dot{w}_o\, L_x L_y \sin c(\frac{k_x L_x}{2}) \sin c(\frac{k_y L_y}{2}) \qquad (1)$$

Substituting $L_x = 3L_y$

$$\dot{W}(k_x, k_y, 0) = \dot{w}_o \frac{L_x^{\ 2}}{3} \sin c(\frac{k_x L_x}{2}) \sin c(\frac{k_y L_x}{6}) \qquad (2)$$

(b) Using equation (2.86), directivity function,

$$D(\theta, \phi) = -i\rho_o ck\, \dot{w}_o \frac{L_x^{\ 2}}{6\pi} \sin c(\frac{kL_x}{2} \sin\theta \cos\phi) \sin c(\frac{kL_x}{6} \sin\theta \sin\phi) \quad (3)$$

(c) Equation (2) can be rearranged in the normalized form,

$$20\log_{10}[\frac{|\dot{W}|}{\dot{w}_o\, L_x^{\ 2}}] = \frac{\sin c(\frac{k_x L_x}{2}) \sin c(\frac{k_y L_x}{6})}{3} \qquad (4)$$

The normalized expression, $20\log_{10}[\frac{|\dot{W}|}{\dot{w}_o L_x^2}]$, is plotted against k_xL_x *and* k_yL_x values for the range from -40 to 40 on the plot as shown in Fig. 1. A cutoff level of -40dB was applied to the plot.

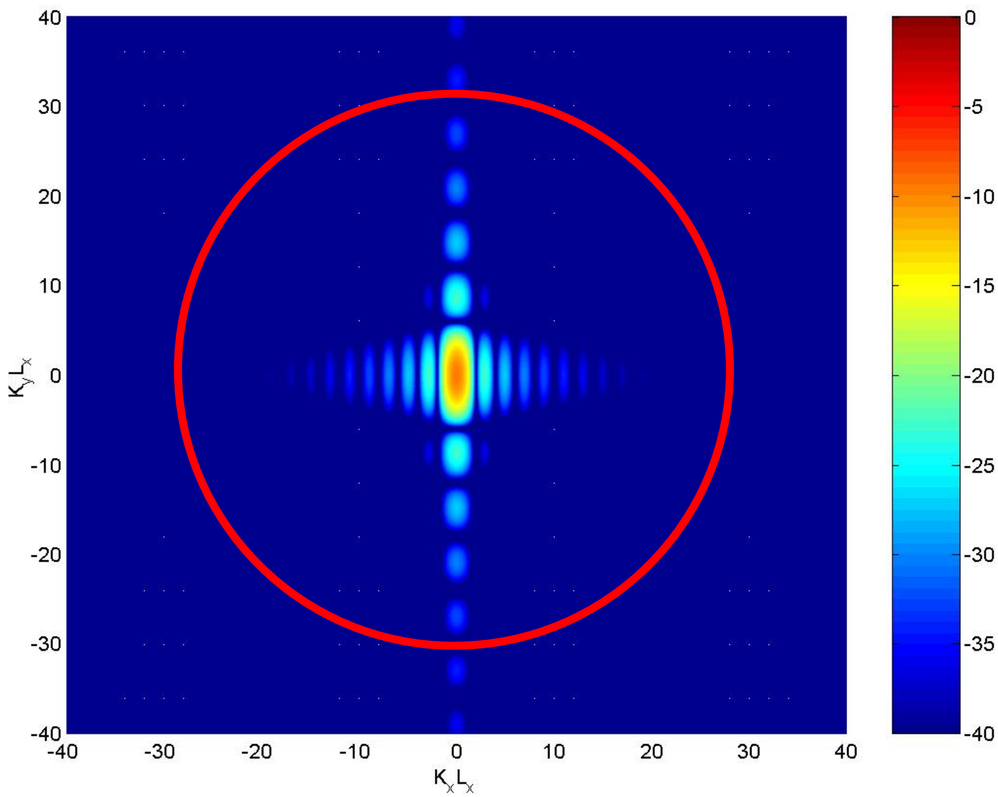


Fig 1. Plot of $20\log_{10}[\frac{|\dot{W}|}{\dot{w}_o L_x^2}]$ against k_xL_x *and* k_yL_x

For a normalized frequency of $kL_x = 30$, a circle with radius of 30 is drawn on the plot. The directivity function can be sketched out using Ewald sphere construction with a hemisphere on a 3-D plot of the contour plot as shown on Fig. 2 and the circle as equatorial base. Note that Fig.2 is drawn on a linear scale.

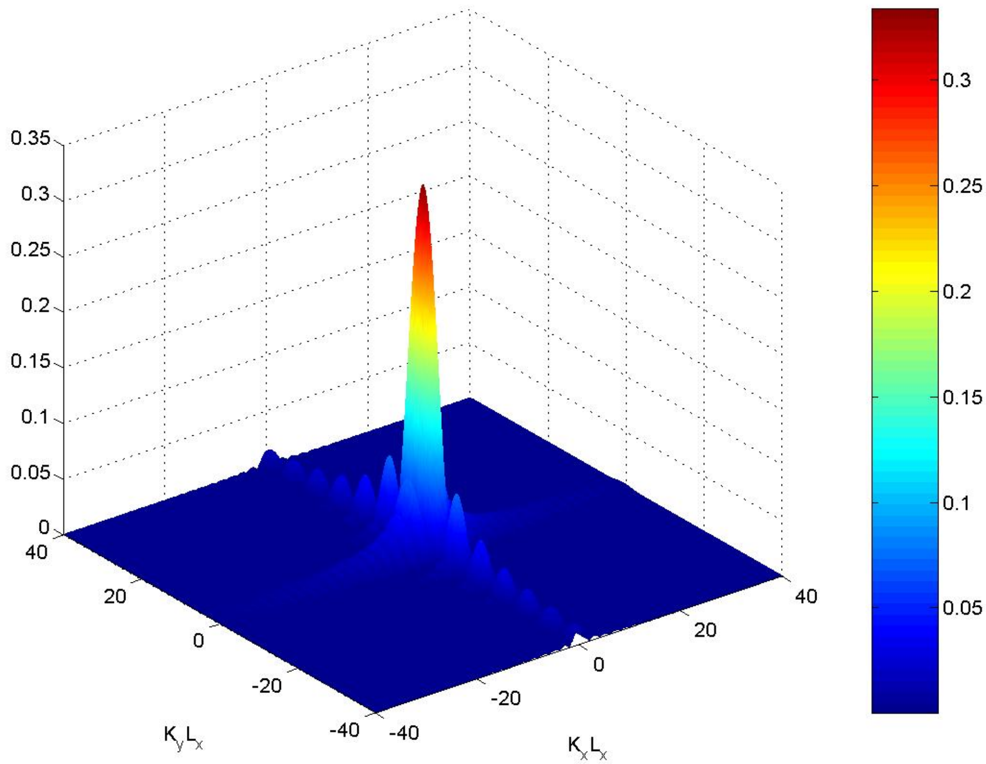
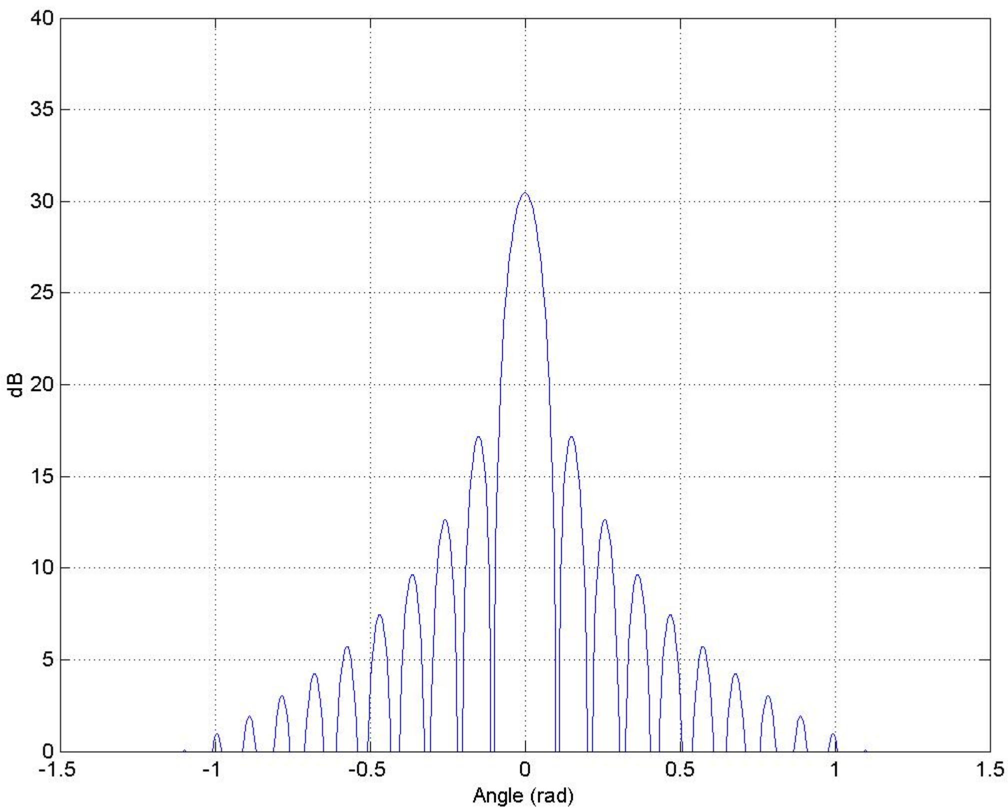


Fig.2 3-D contour plot of $\dfrac{\left|\dot{W}\right|}{\dot{w}_o L_x^{\,2}}$ of square piston

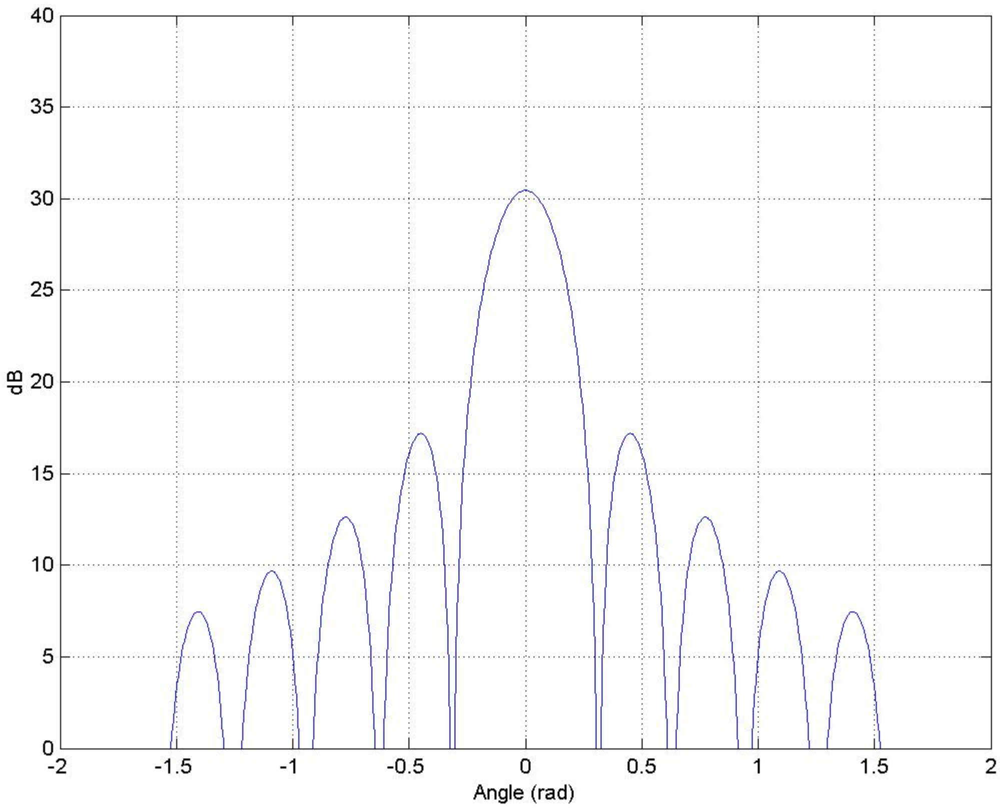
In the x-direction, the number of lobes with level more than -40dB is less as the directivity function contracts towards center since the plate is longer in that dimension and the sinc function decays more rapidly with angle θ. On the other hand, in the y-direction, the lobes decay less rapidly.

2 plots are shown in Fig.3. Fig.3(a) shows the polar plot of the directivity pattern $20\log[\frac{|\dot{W}|}{\dot{w}_o L_x^2}]$ in the x-direction (i.e. $\varphi = 0^o$) reference to 40dB for $-\pi/2 \le \theta \le \pi/2$.

Fig.3(b) shows the polar plot of the directivity pattern $20\log[\frac{|\dot{W}|}{\dot{w}_o L_x^2}]$ in the y-direction (i.e. $\varphi = 90^o$) reference to 40dB $-\pi/2 \le \theta \le \pi/2$.



(a)



(b)

Fig.3. Polar plot of the directivity pattern $20\log[\frac{|\dot{W}|}{\dot{w}_o L_x{}^2}]$ reference to 40dB (cutoff at 0dB)

for $-\pi/2 \leq \theta \leq \pi/2$ in the

(a) x-direction (i.e. $\varphi = 0^o$)

(b) y-direction (i.e. $\varphi = 90^o$)

Problem 2

(a)

Total field in the water halfspace, ψ is given by

$$\psi = (A_i e^{ik_z z} + A_o e^{-ik_z z}) e^{ik_x x} e^{-i\omega t} \tag{1}$$

Thus, total pressure, P_o at z = 0 is

$$P_o = \rho\omega^2 \psi\big|_{z=0}$$

$$P_o = \rho\omega^2 (A_i + A_o) e^{ik_x x} e^{-i\omega t} \tag{2}$$

Total displacement, w_o at z = 0 is

$$w_o = \left.\frac{\partial\psi}{\partial z}\right|_{z=0} = ik_z (A_i - A_o) e^{ik_x x} e^{-i\omega t} \tag{3}$$

Using boundary condition, $P_o = K \bullet w_o$

$$\rho\omega^2 (A_i + A_o) = Kik_z (A_i - A_o) \tag{4}$$

$$A_i = \frac{Kik_z - \rho\omega^2}{Kik_z + \rho\omega^2} A_i$$

The reflection coefficient at the interface, R is thus given by

$$R = \frac{Kik_z - \rho\omega^2}{Kik_z + \rho\omega^2} \tag{5}$$

(b)

As stiffness of the elastic foundation, $K \rightarrow 0$ (i.e. an infinitely flexible foundation), $R \rightarrow -1$. This is similar to the R over a pressure release boundary with a lower vacuum halfspace. On the other hand, as stiffness of the elastic foundation, $K \rightarrow \infty$ (i.e. an infinitely stiff foundation), $R \rightarrow 1$. This is similar to the R over a rigid boundary with a solid lower halfspace.

As frequency of wave, $\omega \rightarrow \infty$, $R \rightarrow -1$. This is similar to the R over a pressure release boundary with a lower vacuum halfspace. This is analogous to vibration at high frequency where the inertia term dominates over stiffness. In the context of this question, the plate is massless but the water has a density of ρ.