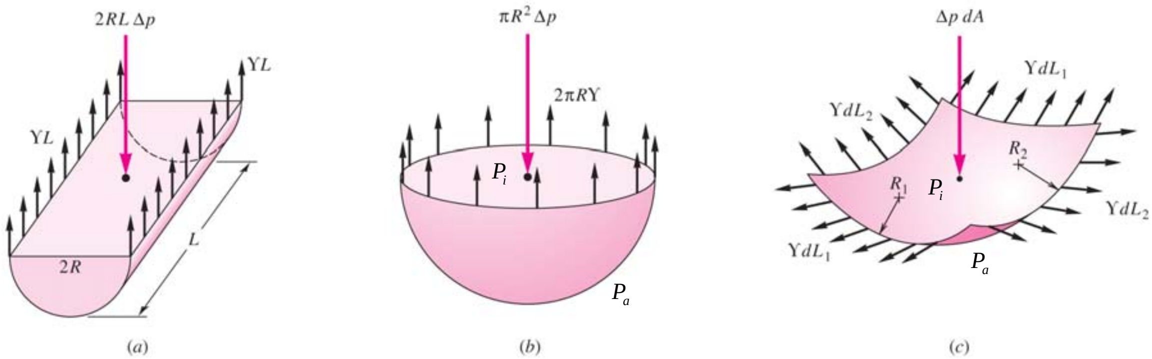
Supplementary note on surface tension

The following figure is from White (e7), Fig. 11 in page 33



In this figure, the surface tension is notated by Υ which is σ in the lecture.

(b) is the same figure for a drop discussed in the lecture

The net force balance is

$$P_i A - P_a A = \sigma 2\pi R$$

$$\Delta P = P_i - P_a = \frac{2\sigma}{R}$$

(c) is a more a general case

The force by pressure differences ΔP $(= P_i - P_a)$ is

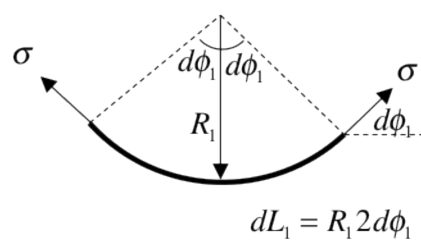
$$F_{\Delta P} = (P_i - P_a) dL_1 dL_2$$

Note, here the area, $dL_1 dL_2$ is not exactly the projected area, perpendicular to the direction in which we consider the force balance. But, as we consider a very small element, the area would be very close to $dL_1 dL_2$.

And, $dL_1 = R_1 2d\phi_1$ and $dL_2 = R_2 2d\phi_2$

Now,

$$F_{\Delta P} = (P_i - P_a) dL_1 dL_2 = (P_i - P_a)(R_1 2d\phi_1)(R_2 2d\phi_2)$$



The force by the surface tension is

$$F_\sigma = 2(\sigma dL_1 \sin d\phi_2) + 2(\sigma dL_2 \sin d\phi_1)$$

Since $d\phi_1$ is very small ($d\phi_1 \ll 1$), $\sin d\phi_1 \approx d\phi_1$

Similarly, $\sin d\phi_2 \approx d\phi_2$

Then, the force by surface tension becomes

$$F_\sigma = 2(\sigma dL_1 \sin d\phi_2) + 2(\sigma dL_2 \sin d\phi_1) = 4\sigma R_1 d\phi_1 d\phi_2 + 4\sigma R_2 d\phi_2 d\phi_1$$

Net force balance gives

$$F_{\Delta P} = F_\sigma$$

$$(P_i - P_a) 4 R_1 R_2 d\phi_1 d\phi_2 = 4\sigma d\phi_1 d\phi_2 (R_1 + R_2)$$

$$P_i - P_a = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$