

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

2.04A Systems and Controls
Spring 2013

Quiz 1

Tuesday, February 26, 2013

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DURATION: 60min (13:05–14:05)

TOTAL PAGES: 3

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1. (40%) We consider a system with input $y(t)$, output $x(t)$, and transfer function

$$\frac{X(s)}{Y(s)} = \frac{s}{(s+2)(s+4)},$$

where $Y(s)$ and $X(s)$ are the Laplace transforms of $y(t)$ and $x(t)$, respectively.

- 1.a) Draw the poles and zeros of this system in the complex plane.
- 1.b) If the input $y(t)$ is a unit step function, what is the output $x(t)$?
- 1.c) If the input $y(t)$ is an impulse function (δ -function), what is the output $x(t)$?
- 1.d) Would you classify the step and impulse responses from the previous two questions as undamped, underdamped, critically damped, or overdamped?
- 1.e) If we instead consider a new system with output

$$w(t) = \int_0^t x(t') dt',$$

what is the new transfer function $W(s)/Y(s)$, and what is the impulse response?

2. (40%) A “conservation law” for population growth in a society is the following:

$$(\text{growth rate}) = (\text{immigration and births rate}) - (\text{emigration and deaths rate}).$$

Let $p(t)$ denote the population at a time instance t . To build a very simple model, let us assume that the immigration rate $m(t)$ is independent of the population, whereas the births, emigration and deaths rates are proportional to the population according to

$$\begin{aligned}\text{births rate} &= bp(t), \\ \text{emigration rate} &= kp(t), \\ \text{deaths rate} &= dp(t).\end{aligned}$$

2.a) Based on the model described above, write down a 1st-order ordinary differential equation that relates the population $p(t)$ with its derivative $\dot{p}(t)$ (the derivative $\dot{p}(t)$ is the population growth rate).

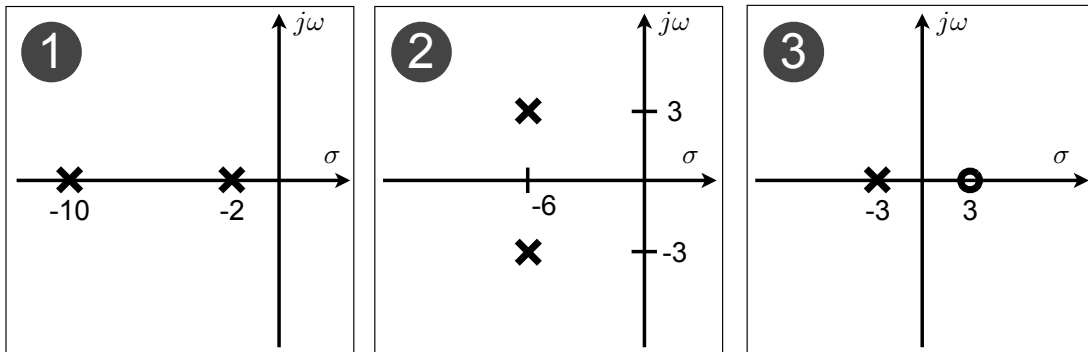
2.b) Further, let us assume that at time $t = 0$ the population is zero, at which instance a constant immigration rate m_0 starts. Moreover, we assume that the births rate is smaller than the combined emigration and deaths rate, *i.e.* $b < k + d$. What is the steady-state population?

2.c) Under the assumptions of the previous question, how long will it take for the population to reach 63.2% of its steady-state value?

2.d) Let us now instead assume that the births rate exceeds the combined emigration and deaths rate, *i.e.* $b > k + d$. How would the population grow then? Is there a steady-state value?

[The model resulting from the last question is attributed to Malthus, 1798.]

3. (20%) A customer requests from you a system whose step response should be as fast as possible and with as little overshoot as possible. You may select from among systems (1), (2), (3) whose complex poles and zeros are shown, respectively, below.



Without explicitly computing their step responses, briefly debate the advantages and disadvantages of each choice (1 ~ 2 sentences per system.)

GOOD LUCK!

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