Dynamics

Dynamics: Kinematics and kinetics of particles, rigid bodies and Continua

Kinematics: Studies motion without its Cause

Kinetics: relates forces and torques to motion

Foundations of Dynamics Newton's laws (axioms)

I. The existence of inertial frame

a free particle stays fix or moves uniformly along a line

II. In an inertial frame, "$F = ma$"

III. Action & Reaction forces are equal & act in opposite directions

This class applies these laws to . particles $\overset{m}{\bullet}$

. Systems of particles

. Rigid bodies / Systems of Rigid bodies

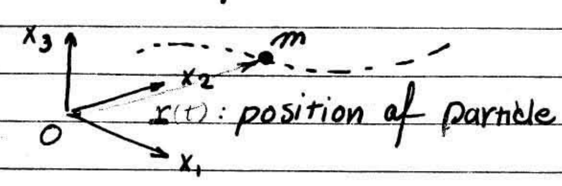
Two approaches: 1) Newton-Euler approach (vectorial) $\Rightarrow$ reaction forces

2) Lagrangian-Hamiltonian approach (Scalar)

$\Rightarrow$ equations of motion

(I) Newton-Euler mechanics
(Newtonian)

(1) Dynamics of a particle



Velocity: $\underline{v}(t) = \dot{\underline{r}}(t) = \frac{d}{dt}\underline{r}(t)$

$\underline{a}(t) = \dot{\underline{v}}(t) = \ddot{\underline{r}}(t)$

$[x_1, x_2, x_3]$ is an inertial frame

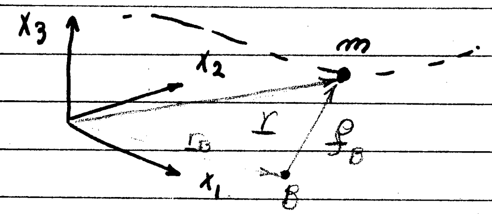
(a) Linear momentum principle

Define: $\underline{P} = m\underline{v}$ linear momentum

Newton II $\Rightarrow$ $\dot{\underline{P}} = \underline{F}$ resultant Force

if $\underline{F} = \underline{0}$ $\Rightarrow$ $\underline{P} = $ Const Conservation of linear momentum

(b) Angular momentum principle

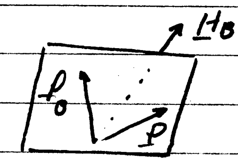


B Can move.
B: a point in $[x_1, x_2, x_3]$ frame (potentially moving)

Define: $\underline{H}_B = \underline{\rho}_B \times \underline{P}$

In words $\underline{H}_B$ is the moment of the lin. momentum w.r.t. B

Also define $\underline{M}_B = \underline{\rho}_B \times \underline{F}$



(resultant torque w.r.t. B)

$$\dot{\underline{H}}_B = \frac{d}{dt}(\underline{\rho}_B \times \underline{P}) = \dot{\underline{\rho}}_B \times \underline{P} + \underline{\rho}_B \times \dot{\underline{P}}$$

$$= (\dot{\underline{r}} - \dot{\underline{r}}_B) \times \underline{P} + \underbrace{\underline{\rho}_B \times \underline{F}}_{\underline{M}_B}$$

$$= -\dot{\underline{r}}_B \times \underline{P} + \underline{M}_B \quad \text{because } (\dot{\underline{r}} \parallel \underline{P})$$

$$\dot{\underline{H}}_B + \underline{v}_B \times \underline{P} = \underline{M}_B$$

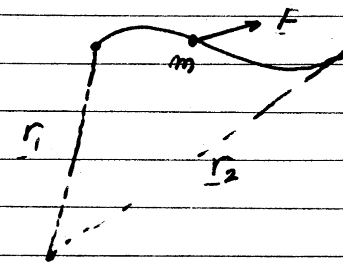
(*) ($\dot{\underline{H}}_B = \underline{M}_B$ if $\underline{v}_B = 0$ or $\underline{v}_B \parallel \underline{P}$ and the second includes the first)

If $\underline{M}_B = \underline{0}$ AND (*) holds then $\underline{H}_B = $ Const

Conservation of angular momentum

(c) Work-Energy Principle

Define $W_{12} = \int_{\underline{r}_1}^{\underline{r}_2} \underline{F} \cdot d\underline{r}$



work done by resultant force

$d\underline{r} = \underline{v}\, dt$

$\underline{F} = m\dot{\underline{v}} \Rightarrow W_{12} = \int_{t_1}^{t_2} m \dot{\underline{v}} \underline{v}\, dt$

$$= \int_{t_1}^{t_2} \frac{d}{dt}\Big(\frac{1}{2} m \underbrace{\underline{v} \cdot \underline{v}}_{|\underline{v}|^2}\Big)\, dt$$

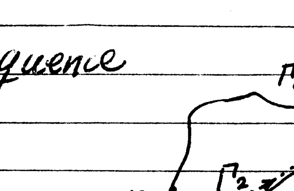
$$= \frac{1}{2} m\, |\underline{v}_2|^2 - \frac{1}{2} m\, |\underline{v}_1|^2$$

Define $T = \frac{1}{2} m |\underline{v}|^2$ Kinetic Energy

$\Rightarrow W_{12} = T_2 - T_1$

(work by F equals to change in kinetic Energy)

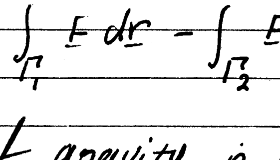
Assume that particle moves in a Force field $\underline{F}(x, \dot{x}) = \underline{F}(x)$, such that



$\int_{\underline{r}_1}^{r_2} \underline{F} \cdot d\underline{r}$ is independent of the path between $\underline{r}_1$ & $\underline{r}_2$

Then $\underline{F}(x)$ is called Conservative.

Consequence



Γ: closed curve

$\Gamma = \Gamma_1 \cup (-\Gamma_2)$

(Γ_2 is defined in opposite direction)

$$\oint_{\Gamma} \underline{F}\, d\underline{r} = \int_{\Gamma_1} \underline{F}\, d\underline{r} - \int_{\Gamma_2} \underline{F}\, d\underline{r} = \underline{0}$$

$\downarrow$

$\underline{F}(x)$ is Conservative

E.g. field of gravity is a Conservative fields

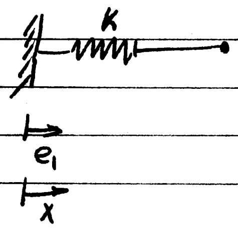
By potential theory, for a Conservative $\underline{F}(x)$, There exists $V(\underline{x})$ (the potential) Such that $\underline{F} = -\nabla V$ (-grad V)

$$= -\left(\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}, \frac{\partial V}{\partial x_3}\right)$$

Eg. gravitational field

$g\downarrow \quad \downarrow y \quad \overset{m}{\downarrow} mg \qquad \underline{F} = m\underline{g} \Rightarrow V = mgy$

Eg. Spring force



$\underline{F} = -kx\underline{e}_1 \Rightarrow$ potential $V = \frac{1}{2}kx^2$

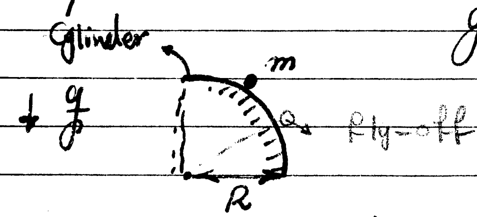
$$\Rightarrow \quad W_{12} = \int_{\underline{r}_1}^{\underline{r}_2} \underline{F}\, d\underline{r} = \int_{\underline{r}_1}^{\underline{r}_2} (-\nabla V)\, d\underline{r} = V_1 - V_2$$

$$\Rightarrow T_2 - T_1 = V_1 - V_2$$

$$\Rightarrow T_1 + V_1 = T_2 + V_2$$

$\Rightarrow E = T + V$ total mechanical Energy in Conserved in a potential force field

Example: point mass slides on cylinder under the effect of gravity.



- what is the fly-off angle?
- How does φ^* depend on R & m?