37 Nyquist Plot

Consider the attached images; here are a few notes. In the plant impulse response, the initial condition before the impulse is zero. The frequency scale in the transfer function magnitude plots is $10^{-3} - 10^{4}$ radians per second. In the plot of $P(s)$ loci, the paths taken approach the origin from $\pm 90°$, and do not come close to the critical point at $-1+0j$, which is shown with an `x`. In the plot of $P(s)C(s)$ loci, the unit circle and some thirty-degree lines are shown with dots. Also, the two paths in this plot connect off the page in the right-half plane.

Answer the following questions by circling the correct answer.

1. The overshoot evident in the open-loop plant is about
   (a) 120%
   (b) there is no overshoot since this is not a step response
   (c) **70**%
   (d) 40%

2. The natural frequency in the open-loop plant is about
   (a) one Hertz
   (b) **one radian per second** - To compute this, you need a whole cycle.
   (c) 1.2 radians per second
   (d) six radians per second

3. Based on the plant behavior, $P(s)$ probably has
   (a) no zeros and one pole
   (b) one zero and one pole
   (c) no zeros and two poles
   (d) **one zero and two poles** - This plant has a zero at +1 (yes, a right-half plane zero, also known as an unstable zero) and two poles at $-0.1 \pm j$. You can tell it has two complex,stable poles because of the ringing in this impulse response. You can tell it has a zero because the output instantaneously moves to a nonzero value during the impulse - this could only be caused by a differentiator.

4. Compare the abilities of the plant hooked up in a unity feedback loop (i.e., with $C(s) = 1$), and of the designed closed-loop system, to follow low-frequency commands:
   (a) The $P(s)C(s)$ case has a lot more magnitude above one radian per second, and so it has a better command-following
   (b) $P(s)$ is nice and flat at low frequencies, so it is better at command-following

(c) **P(s)C(s) has increasing values at lower frequencies and this makes it better** - Setting $C(s) = 1$ will achieve about 10% tracking accuracy at low frequencies. The designed $P(s)C(s)$ has a pole at or near the origin and hence is an integrator; this gives us no tracking error in the steady state.

The plant is stable, but lightly damped and it has an unstable zero. As you can guess from a quick check with a root locus, this is a difficult control problem, intuitively because the plant always moves in the wrong direction first. A PID cannot stabilize this system! I ended up using the loopshaping method in MATLAB's LTI design tool; this gave a third-order controller with two zeros.

(d) The peak in $P(s)$ is not shared by the other plot and this makes $P(s)$ better at command-following.

5. Is the unity feedback loop stable, based on the loci of $P(s)$?

(a) **No: The path encircles the critical point once in the clockwise direction and that is all it takes, because the poles of $P(s)C(s)$ are in the left-half plane** - Note that the unstable zero in the plant is immaterial by itself. Nyquist's rule is that stability is achieved if and only if $p = ccw$, where p is the number of unstable poles in $P(s)C(s)$, and ccw is the number of counter-clockwise encirclements of the critical point.

(b) Yes: The path encircles the critical point once in the counter-clockwise direction

(c) Yes: The path encircles the critical point once clockwise and this is matched by a plant zero in the right-half plane

(d) No: The path encircles the critical point twice whereas it should only circle it once.

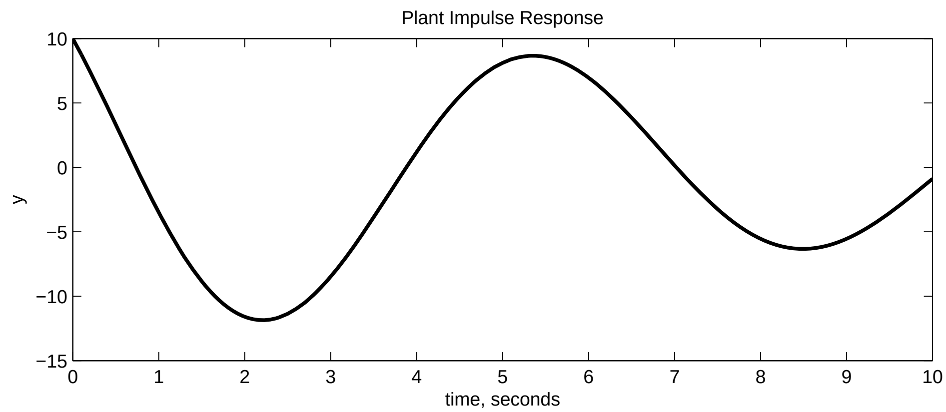
6. The designed compensator creates a stable closed-loop system, as is seen in the step response plot. The gain and phase margins achieved are approximately:

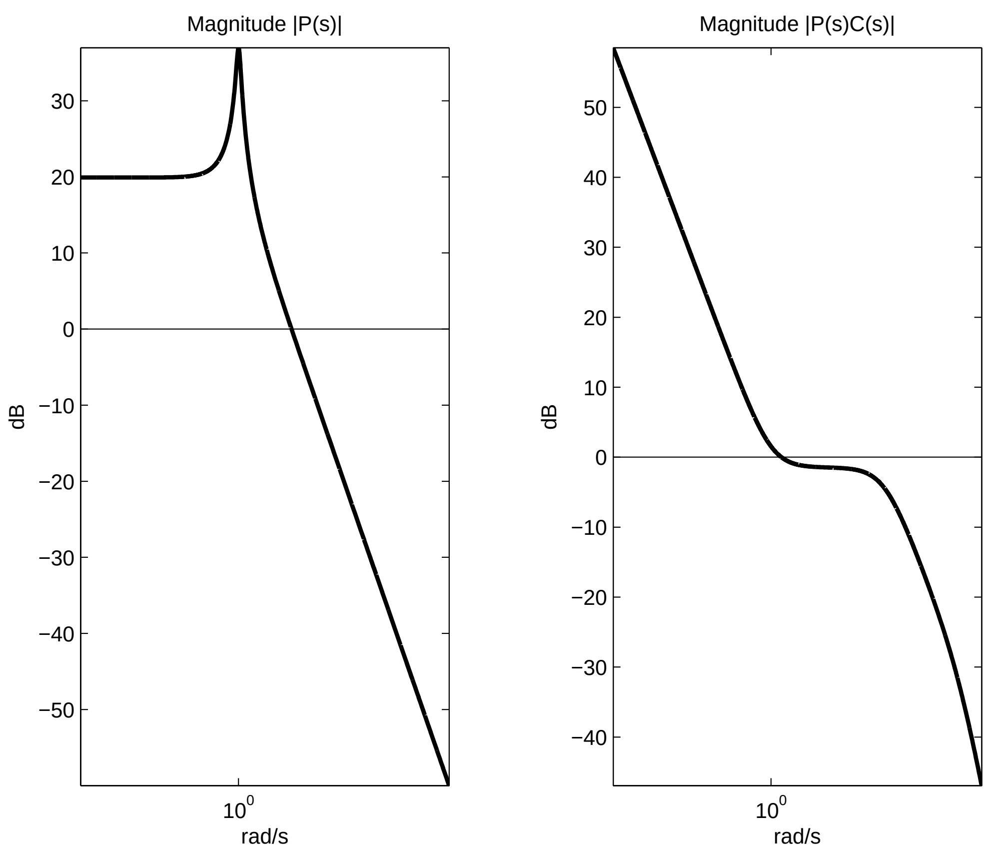
(a) 0.2 upward gain margin, 1.2 downward gain margin, and $\pm 40°$ phase margin

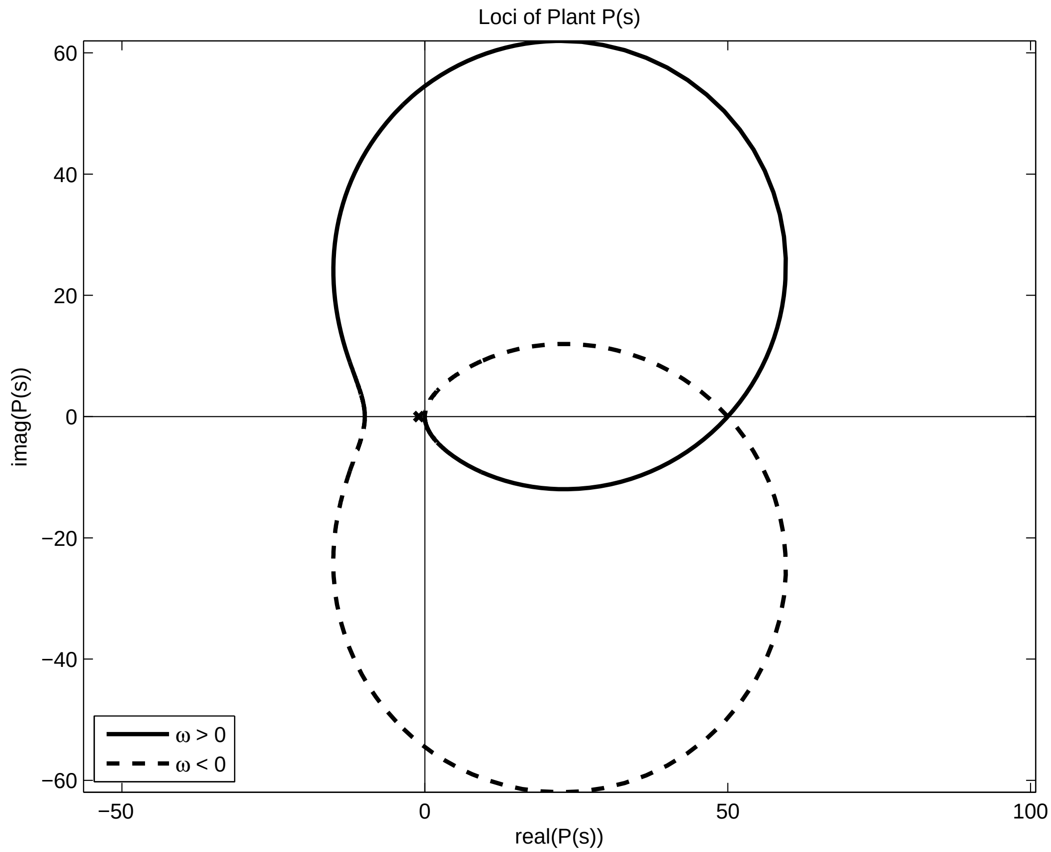
(b) **1.2 upward gain margin, infinite downward gain margin, and $\pm 30°$ phase margin**

(c) infinite upward gain margin, 1.2 downward gain margin, and $\pm 30°$ phase margin

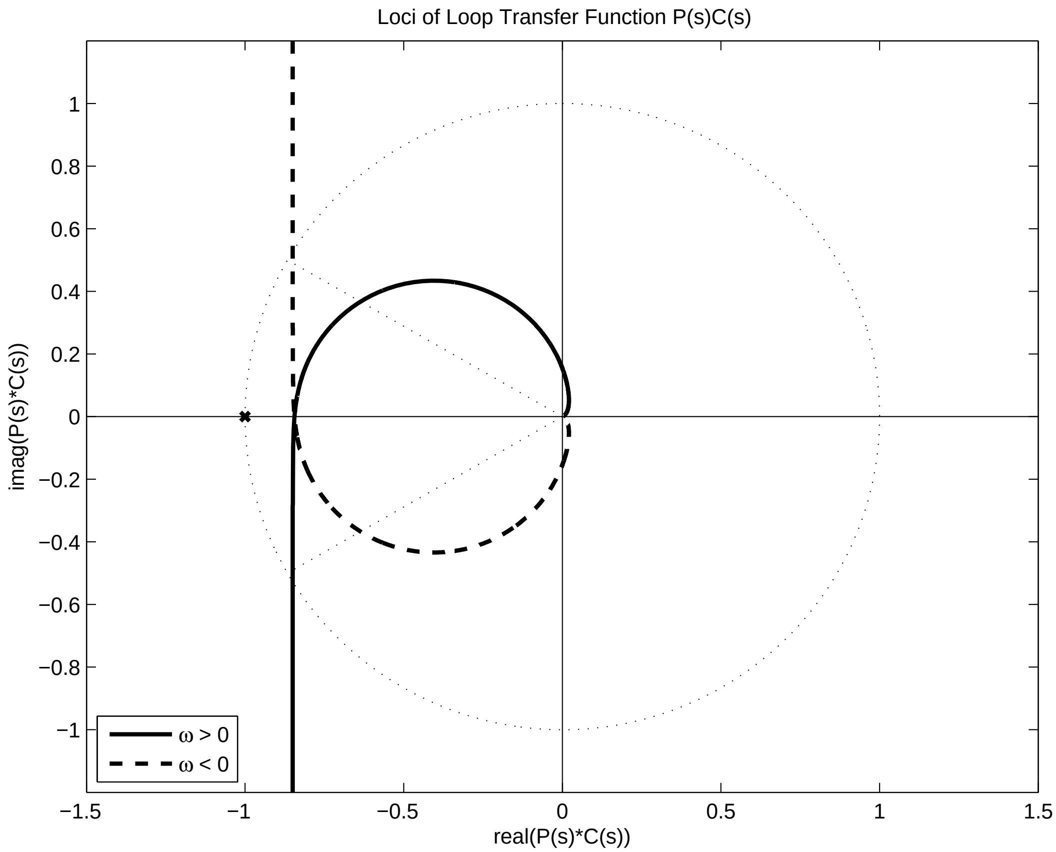
(d) 1.2 upward gain margin, infinite downward gain margin, and $\pm 60°$ phase margin

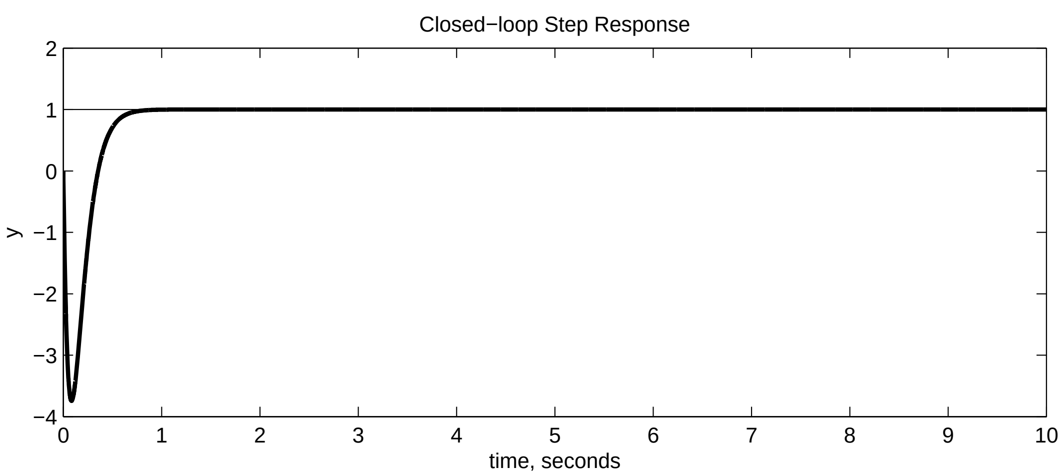

Plant Impulse Response
y
time, seconds
10
5
0
−5
−10
−15
0 1 2 3 4 5 6 7 8 9 10


Magnitude |P(s)|
dB
rad/s
30
20
10
0
−10
−20
−30
−40
−50
10^0
Magnitude |P(s)C(s)|
dB
rad/s
50
40
30
20
10
0
−10
−20
−30
−40
10^0



Loci of Plant P(s)
60
40
20
0
−20
−40
−60
imag(P(s))
−50
0
50
100
real(P(s))
ω > 0
ω < 0


Loci of Loop Transfer Function P(s)C(s)
imag(P(s)*C(s))
real(P(s)*C(s))
ω > 0
ω < 0
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.5
−1
−0.5
0
0.5
1
1.5


Closed−loop Step Response
y
time, seconds
2
1
0
−1
−2
−3
−4
0
1
2
3
4
5
6
7
8
9
10