13.012 Marine Hydrodynamics for Ocean Engineers
Fall 2004 Quiz #1

Student name: SOLUTIONS

This is a closed book examination. You are allowed 1 sheet of 8.5" x 11" paper with notes.

For the problems in Section A, fill in the answers where indicated by ____________, or in the provided space. When a list of options is presented ([...],[...],[....] etc), circle all the options (all, none, one or more) that apply.

Use the following constants unless otherwise specified:

Gravity: $g = 10\ m/s^2$ water density:	$\rho_w = 1000\ kg/m^3$	kinematic viscosity: $\nu_w = 1\times10^{-6}\ m^2/s$
Seawater density:	$\rho_w = 1025\ kg/m^3$	kinematic viscosity: $\nu_{sw} = 1\times10^{-6}\ m^2/s$
Air density:	$\rho_a = 1\ kg/m^3$	kinematic viscosity: $\nu_a = 1\times10^{-5}\ m^2/s$

Assume the fluid is incompressible unless otherwise defined.
Give all answers in SI units (kg, m, s). All numerical answers MUST have the proper units attached.

Part A (30%):

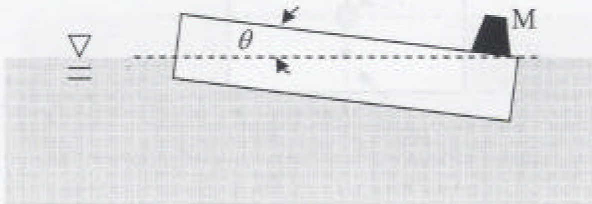
1) A 1 m^3 block of aluminum, specific gravity 2.7, is tethered to a piece of cork, specific gravity 0.24. The volume of cork required to keep the block neutrally buoyant in seawater is 2.117 m^3 and the volume required in fresh water is 2.24 m^3. Assume both the aluminum and cork are fully submerged and that the weight of the tether is negligible.

2) The velocity field $\vec{V} = 4xy\,\hat{i} + 10y^2\,\hat{j} + C\,zy\,\hat{k}$ is valid for C = -24. The vorticity in this flow field is $\omega = \nabla \times \vec{V} = -24z\hat{i} - 4y\hat{k}$. This flow field is [irrotational][rotational].

3) The two *linearized* boundary conditions at the free surface for linear progressive free-surface gravity waves are $\eta = -\frac{1}{g}\frac{\partial\phi}{\partial t}$ and $\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0$ on $z=0$ (give mathematically).

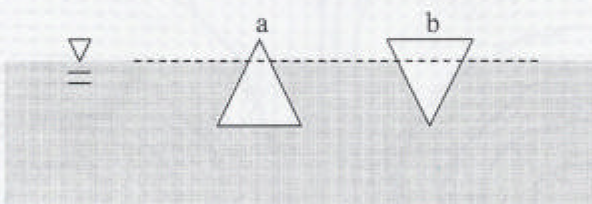
4) The linear free surface dispersion relationship is given by $\omega^2 = gk\tanh(kH)$. This relationship holds for [shallow] [intermediate] [deep] [all of the above] waves. The shallow water form of the dispersion relation is $\omega^2 = gk^2H$. This holds for H/λ [greater than] [less than] 1/20.

5) When a $M=5kg$ weight is placed at the end of a uniform density floating wooden beam ($3\ m$ long, with a $10cm$ x $10\ cm$ cross section), the beam tilts at an angle θ such that the upper right corner of the beam is exactly at the surface of the water (as shown below). The angle θ = 0.1° and the specific gravity of the log is 0.985



6) A progressive linear free surface gravity wave train is propagating in a tank from left to right. This wave train is generated by a wave paddle at one end. The deep water wave has frequency $\omega = 1\, rad/s$. The time it takes for the front edge of the wave packet to reach the far end of the tank, 100 meters away, is 20 seconds.

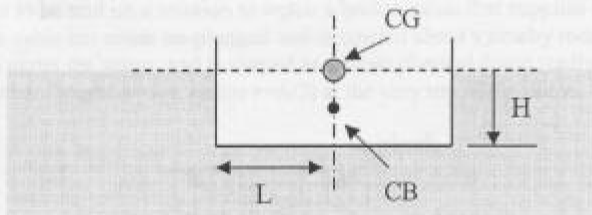
7) When floating in water ($\rho = 1000 kg/m^3$), an equilateral triangular body (SG = 0.9) with a long length into the board is more stable in position [a] [b].



8) A deep water wave train with amplitude $a = 0.5m$ is propagating from left to right in a tank. The tank depth is 15 meters, and the wave frequency is 0.25 Hz. These waves are [linear] [non-linear][cannot determine] water waves. The appropriate dispersion relationship is given by ω^2 = gk. The wavelength λ = 25 m. Phase speed V_p = 6.28 m/s. Group speed of the waves, V_g = 3.14 m/s.

9) For the rectangular barge with width *2L* and vertical draft *H* shown below, the Metacentric Height $\overline{GM}$ = $\frac{L^2}{3H} - \frac{H}{2}$ for small tilt angles (in terms of *L* and *H*). This barge can only be stable if $\frac{L}{3H}$ > $\frac{H}{2}$.

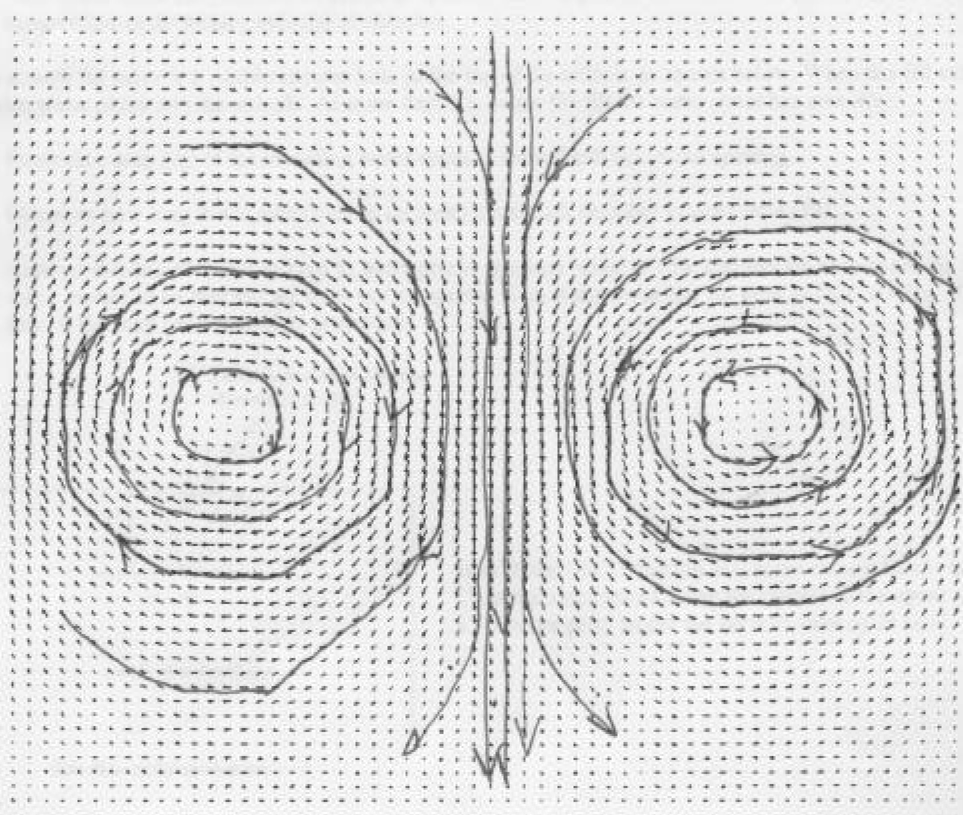
$(\overline{GM} > 0)$



10) The conservation of mass equation depends on the assumption(s) of [constant density] [irrotationality] [inviscid fluid] [incompressibility] [Newtonian fluid] [matter cannot be created].

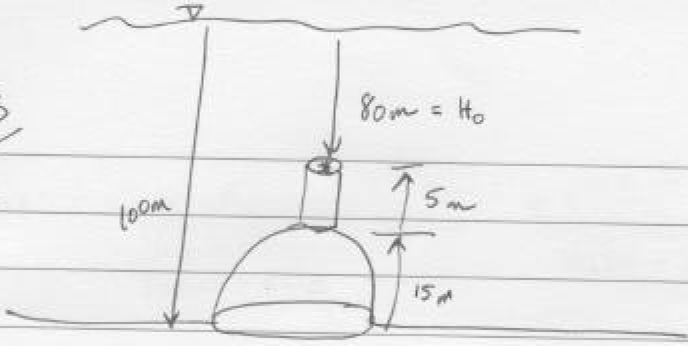
For incompressible flow, the density ρ satisfies the equation $\frac{D\rho}{Dt} = 0$.

11) BONUS (5pts) Sketch *ten* streamlines on the plot below. Vectors represent the flow velocity.



PART B

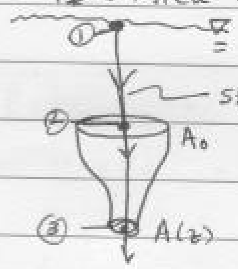
#1



a) $P_* = -\rho g H_o = -\rho g(-80) = 1025 \cdot 10 \cdot 80 = \underline{\underline{8.2 \times 10^5\ Pa}}$

b) $F = P_* \cdot \text{Area} = 8.2 \times 10^5\ Pa * \pi (2.5)^2 = 1.61 \times 10^7\ N$

c)



streamline goes from free surface to top of hatch to A(z)

$$\cancel{p_1} + \tfrac{1}{2}\rho V_1^2 + \rho g z_1 = \cancel{p_2} + \tfrac{1}{2}\rho V_2^2 + \rho g z_2 = \cancel{p_3} + \tfrac{1}{2}\rho V_3^2 + \rho g z_3$$

$P_1 = p_{atm}$! @ surface

$\therefore P_1 = P_2 = P_{atm}$

$V_1 = \emptyset$ @ free surface there is no velocity

also $z_1 = \emptyset$

$$\emptyset = \tfrac{1}{2}\rho V_2^2 + \rho g z_2 = \tfrac{1}{2}\rho V_3^2 + \rho g z_3$$

Also by continuity : $A_o V_2 = A(z) V_3$; $A(z) = \frac{A_o V_2}{V_3}$

$$V_2^2 = \frac{-\rho g(-80)}{\tfrac{1}{2}\rho} = 1600 \quad \boxed{V_2 = 40\ m/s}$$

$$V_3^2 = \frac{-\rho g z_3}{\tfrac{1}{2}\rho} = -2g(-80 - z) = 160g + 20z$$

$$A(z) = \frac{A_o V_z}{\sqrt{1600+20z}} = \frac{\pi\left(\frac{0.01}{2}\right)^2 \cdot 40}{\sqrt{1600+20z}}$$

$$\boxed{\therefore A(z) = \frac{0.001\,\pi}{\sqrt{1600+20z}}}$$

d) fill Rate $\rightarrow V_z = 40\,m/s$

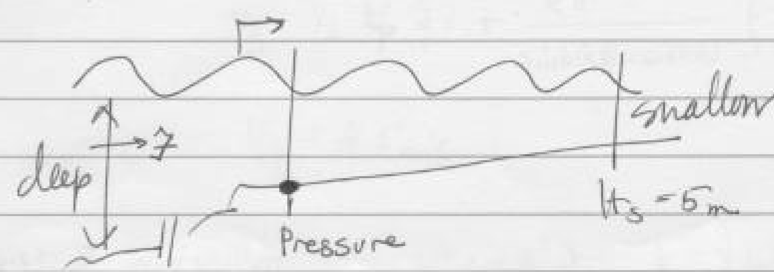
Volume flow Rt $= V_z A_o \Rightarrow \left[\frac{L^3}{T}\right]$

$$A_o = \frac{\pi D_o^2}{4} \qquad \dot{Q} = \frac{40 \cdot \pi \cdot (0.01)^2}{4}$$

$$\text{time} = \frac{V_{ol}}{\dot{Q}} = \frac{\cancel{\pi}(2.5)^2 \cdot 5}{10\cancel{\pi}(0.01)^2} = \underline{31250} \text{ seconds}$$

$\boxed{8.68 \text{ hours}}$

②



$|\Delta P| = 6000\,N/m^2$ $\qquad p = -\rho\frac{\partial\phi}{\partial t} \qquad \eta = \frac{-1}{g}\frac{\partial\phi}{\partial t}\Big|_{z=0}$

$p = \rho g \eta e^{+kz}$

$\cancel{p = \rho g a \cos(kx - \omega t)}$

$\cancel{\rho g a = 3000}$

Waves have $\omega = \frac{2\pi}{T} = 0.628\ \text{rad/s}$; $\lambda = 160\text{m}$ in deep water

but @ β not deep!

so $\omega^2 = gk\tanh(kH)$

$$p(z,t) = \rho g a e^{kz} \cos(kx - \omega t)$$

$z = -30\text{m}$

$$p(z,t) \doteq \rho g a e^{-30k} \cos(kx - \omega t)$$

intermediate depth

$$\frac{\omega^2}{gk} = \tanh(kH)$$

$k \simeq 0.045\ \text{m}^{-1}$

$\boxed{\lambda = 139.6\ \text{m}}$

$$|p(z,t)| = 3000\ \text{N/m}^2 = \rho g a e^{-30(0.045)}$$

(used $\rho = 1000\ \text{kg/m}^3$)

$$\frac{0.3}{e^{-(30 \cdot 0.045)}} = a$$

$\therefore \boxed{a = 1.157\ \text{m}}$

(b) @ Point A $\rightarrow$ $\lambda = 139.6\text{m}$

$a = 1.157\text{m}$

$$V_g = \tfrac{1}{2} V_p \left\{ 1 + \frac{kH}{\sinh(kH)\cosh(kH)} \right\} \qquad V_p = \omega/k$$

$$V_g = 9.5\ \text{m/s}$$

(a) @ point B now shallow! $\omega^2 = gk^2H$ $\quad k = \sqrt{\omega^2/gH}$

$\omega = 0.628\ \text{rad/sec}$

given constant...

$k = 0.089\ \text{1/m}$ $\quad \boxed{\lambda = 70.7\text{m}}$

$\boxed{\text{a should be same as @ pt A}}$

$$V_g = V_p = \frac{\omega}{k} = \frac{0.628}{0.089} = \boxed{7.06\ \text{m/s}} ..$$

$\mathcal{F} = E \cdot V_g$

deep water

$\omega = 0.628 \text{ rad/s}$ $\qquad v_g = \frac{V_p}{2} = \frac{1}{2}\frac{\omega}{k}$

$k = \omega^2/g = 0.0394 \text{ 1/m}$

$\boxed{V_g = 7.97 \text{ m/s}}$ $\qquad E = \frac{1}{2}\rho g a^2 \qquad a = 1.157\text{m}$

$E = 6693.2 \text{ J/m}^2$

$\boxed{\mathcal{F} = 53342.1 \ \frac{J}{s\,m}}$

c) volume transport

$$\dot{m} = \int_{-H}^{0} \rho u \, dz$$

$$\bar{\dot{m}} = \frac{1}{T}\int_0^T \left[\int_{-H}^{0} \rho u \, dz\right] dt$$

$$= \text{Constant} \cdot \underbrace{\frac{1}{T}\int_0^T \sin(\ \ ) \, dt}_{=0}$$

$\boxed{\bar{\dot{m}} = 0 \quad \text{average mass transport}}$

d) $\frac{\text{Vol flux}}{\text{width}} = \frac{V \cdot \text{Area}}{\text{width}} = \int_{-\frac{\lambda}{2}}^{0} u \, dz$

$u = \frac{\partial \phi}{\partial x} \Rightarrow \qquad \frac{\dot{V}}{L} = \int_{-\frac{\lambda}{2}}^{0} \frac{\partial \phi}{\partial x}(z) \, dz$

deep $\phi = \frac{-a\omega}{k} e^{kz} \sin(kx - \omega t)$

$\frac{\partial \phi}{\partial x} = a\omega e^{kz} \cos(kx - \omega t)$ $\qquad$ function of time...

e)