2.00AJ / 16.00AJ Exploring Sea, Space, & Earth: Fundamentals of Engineering Design
Spring 2009

FUNdaMENTALs for Design Analysis: Fluid Effects & Forces

Prof. A. H. Techet

2.00a/16.00a Lecture 4

Spring 2009

Water & Air

Courtesy of the U.S. Navy.

Courtesy of NASA.

- Hydrodynamics v. Aerodynamics
  - *Water is almost 1000 times denser than air!*

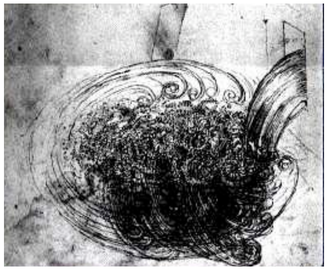


Image by Leonardo da Vinci.

- Air
  - Density

$$\rho = 1.2\, kg / m^3$$

  - Dynamic Viscosity

$$\mu = 1.82 \times 10^{-5}\, N \cdot s / m^2$$

  - Kinematic Viscosity

$$\nu = \mu / \rho \quad = 1.51 \times 10^{-5}\, m^2 / s$$

- Water
  - Density

$$\rho = 1025\, kg / m^3 \text{ (seawater)}$$

$$\rho = 1000\, kg / m^3 \text{ (freshwater)}$$

  - Dynamic Viscosity

$$\mu = 1.0 \times 10^{-3}\, N \cdot s / m^2$$

  - Kinematic Viscosity

$$\nu = 1 \times 10^{-6}\, m^2 / s$$

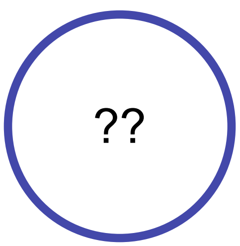
Fluid Properties @20ºC

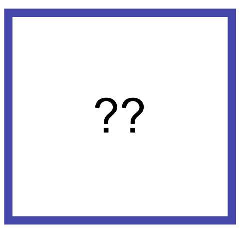
Hydrostatic Pressure

Pressure under water

- Pressure is a Force per Area (P = F/A)

Pressure is a Normal Stress





Pressure is *isotropic.*

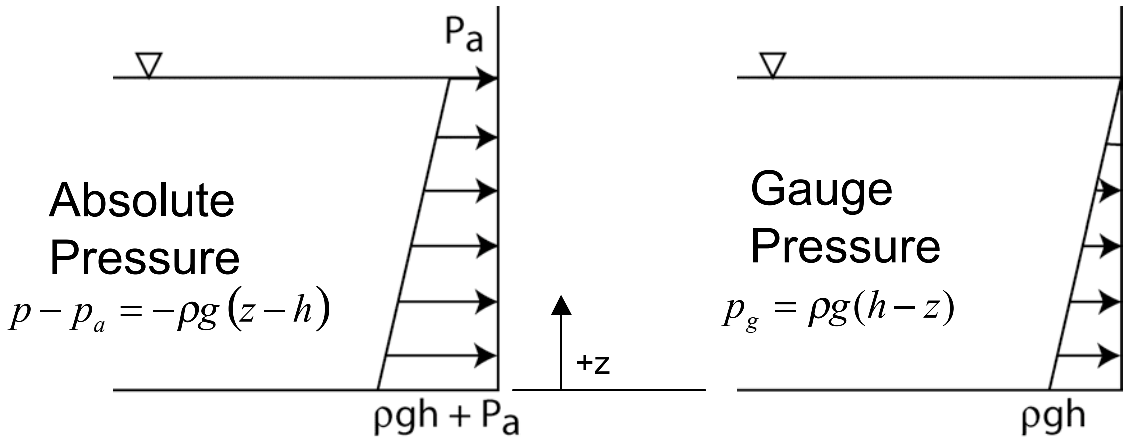
How does it act on these 2D shapes?

Pressure increases with depth

- Hydrostatic Pressure

$$\frac{dp}{dz} = -\rho g$$

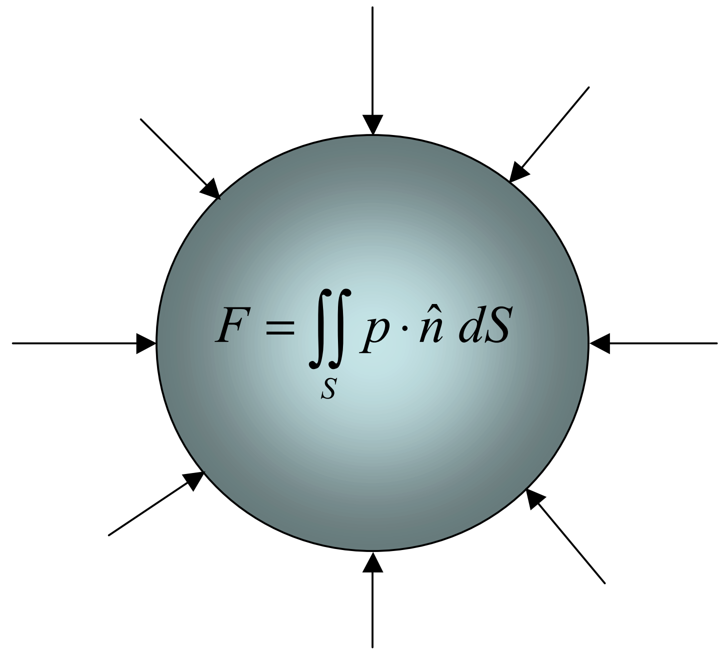
- Pressure on a vertical wall:



The NET pressure force acts at the CENTER of PRESSURE

FOR MORE DETAILS WITH THE DERIVATION OF THE HYDROSTATIC EQUATION, SEE THE READING ON PRESSURE POSTED ON THE CLASS WEBPAGE

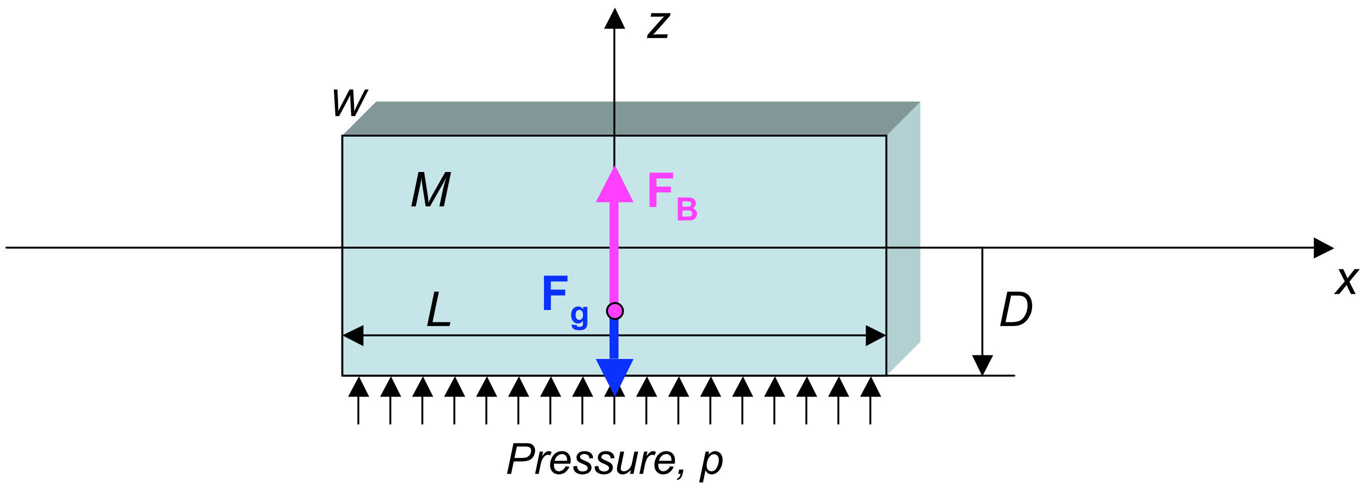
Pressure on a sphere at depth?



Pressure acts normal to the surface. By convention pressure is positive in compression. The *total force* is the integration of the ambient pressure over the surface area of the sphere.

Archimedes' Principle

Weight of the displaced volume of fluid is equal to the hydrostatic pressure acting on the bottom of the vessel integrated over the area.



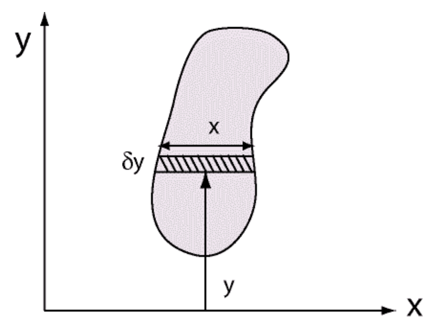
$$F_z = p*A = (\rho g\, D) * (L\, W)$$
$$= \rho g * (L\, D\, W)$$
$$= \rho g * Volume$$

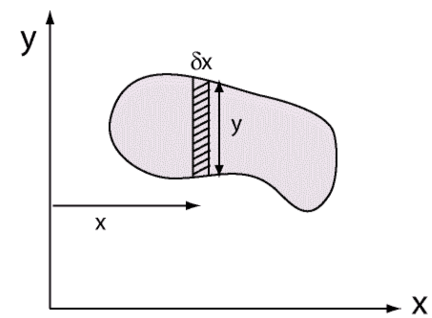
BUOYANCY FORCE -->

Center of Buoyancy

Center of buoyancy is the point at which the buoyancy force acts on the body and is equivalently the geometric center of the submerged portion of the hull.

To calculate the center of buoyancy, it is first necessary to find the center of area!





1) Calculate the Area of the body:

$$A = \int y(x)\, dx$$

2) Find the 1st moment of the area:

$$M_{xx} = \int x(y)\, y\, dy$$

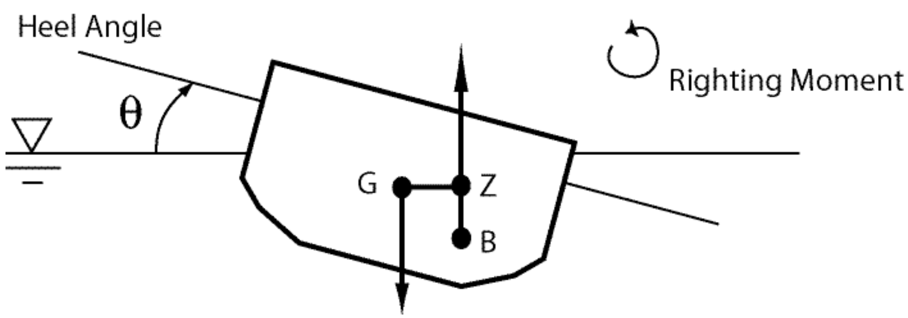
$$M_{yy} = \int y(x)\, x\, dx$$

3) Calculate the coordinates for the Center of Buoyancy:

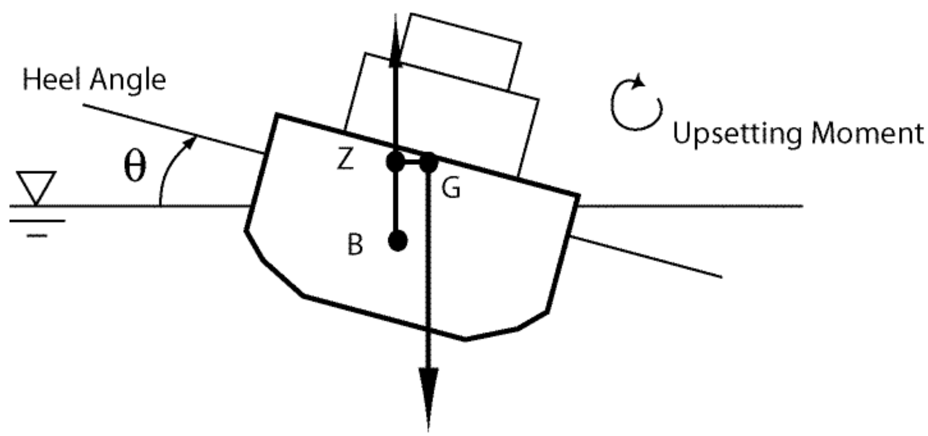
$$\bar{x} = \frac{M_{yy}}{A} \quad \text{and} \quad \bar{y} = \frac{M_{xx}}{A}$$

Stability?

A statically stable vessel with a positive righting arm.



"self-righting"



Statically unstable vessel with a negative righting arm.

Fluids in Motion…

Fluids Follow Basic Laws

- *Conservation of Mass*
- *Conservation of Momentum*
- *Conservation of Energy*

Flows can be described simply

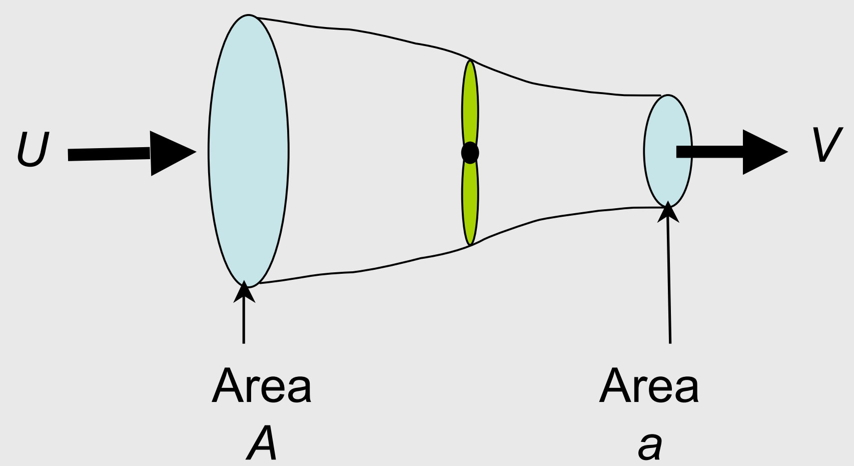
- Streamlines are lines *everywhere* parallel to the velocity (*no* velocity exists perpendicular to a streamline)
- Streaklines are instantaneous loci of *all* fluid particles that pass through point x_o
- Pathlines are lines that one single fluid particle follows in time

In *Steady* flow these are all the same!
Steady flow does not change in time.

Conservation of Mass

What goes in must come out!

Control Volume: nozzle



Mass: *density * volume*

$$m = \rho \forall$$

Volume: *area * length*

$$\forall = A \cdot L$$

Length: *velocity * time*

$$L = U \cdot \Delta t$$

$$\therefore m = \rho A U \cdot \Delta t$$

To conserve Mass:

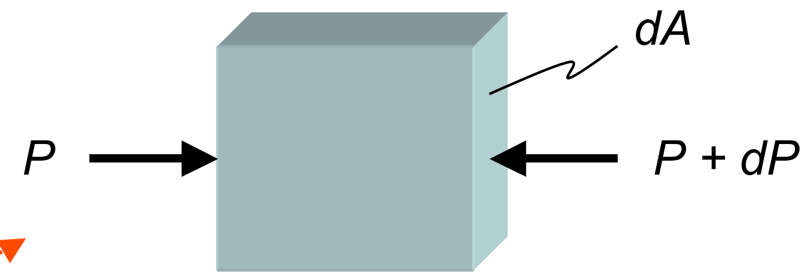
$$m_{in} = m_{out}$$

$$\underbrace{\rho A U \cdot \Delta t}_{m_{in}} = \underbrace{\rho a V \cdot \Delta t}_{m_{out}}$$

Conservation of Momentum

Newton's second law states that *the time rate of change of momentum of a system of particles is equal to the sum of external forces acting on that body*.

$$\Sigma \mathbf{F_i} = \frac{d}{dt}\{m\mathbf{V}\}$$



Forces:

- Gravity (hydrostatic)
- Pressure !!!
- Shear (viscous/friction)
- External Body

$$\Sigma \mathbf{F}_x = \cancel{P}\, dA - \left(\cancel{P} + dP\right) dA = m\frac{dv}{dt}$$

$$\Sigma \mathbf{F}_x = -dP\, dA = m\frac{dv}{dt}$$

Pressure Along a Streamline

Bernoulli's Equation
(neglecting hydrostatics)

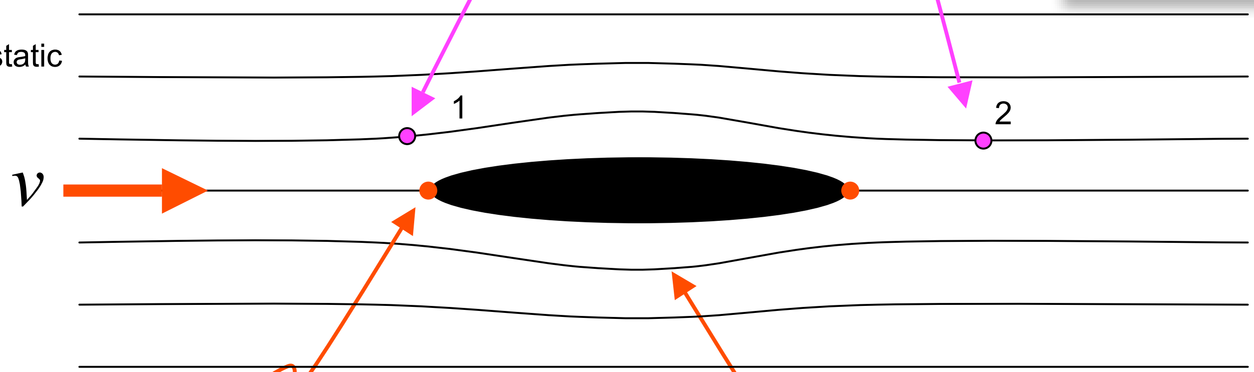
$$P_1 + \frac{1}{2}\rho v_1{}^2 = P_2 + \frac{1}{2}\rho v_2{}^2 = Const$$

General eqn. on streamline:

$$P + \frac{1}{2}\rho v^2 = C$$

Atmospheric pressure

$P_{atm} = P_{static}$



Stagnation point
(high pressure)

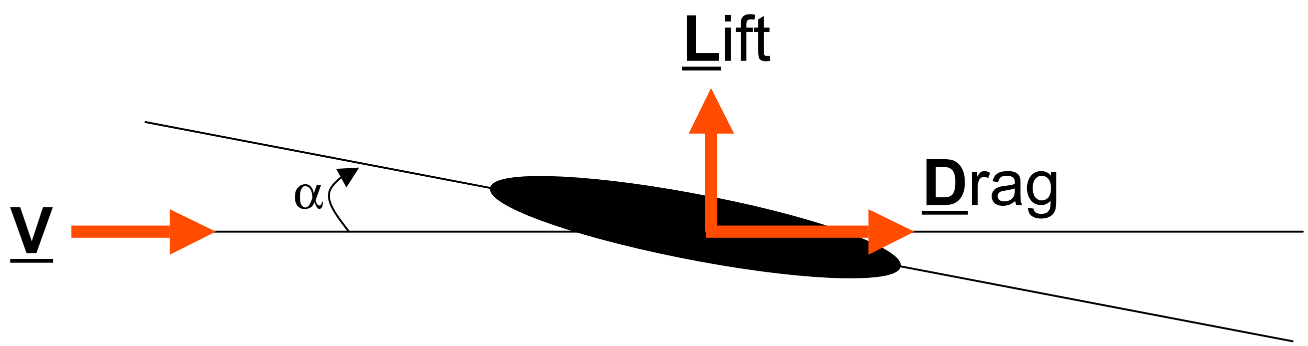
Streamline
(C = constant)

Stagnation pressure:

$$P_o = P_{atm} + \frac{1}{2}\rho v^2$$

NB: The body surface can also be treated as a streamline as there is no flow through the body.

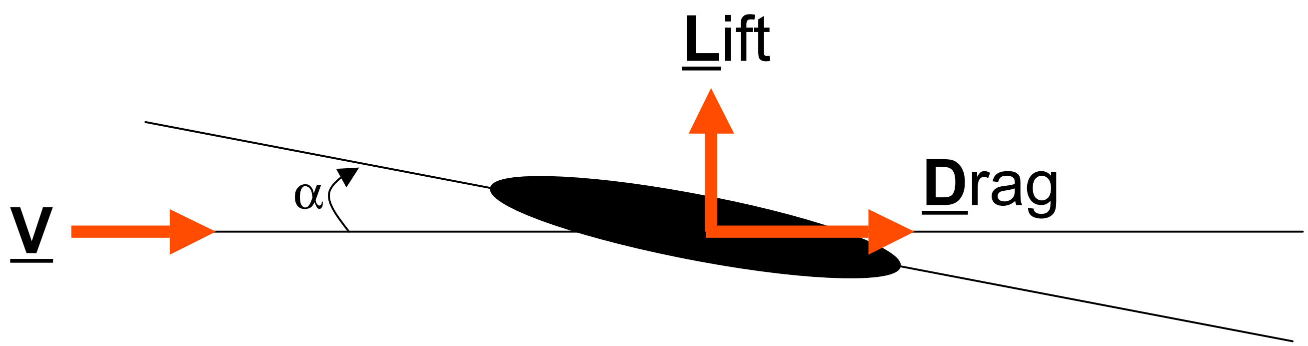
Aero/Hydro-dynamic Forces



$$L = \frac{1}{2}\rho V^2 \cdot C_L \cdot S$$

$$D = \frac{1}{2}\rho V^2 \cdot C_D \cdot S$$

Aero/Hydro-dynamic Forces

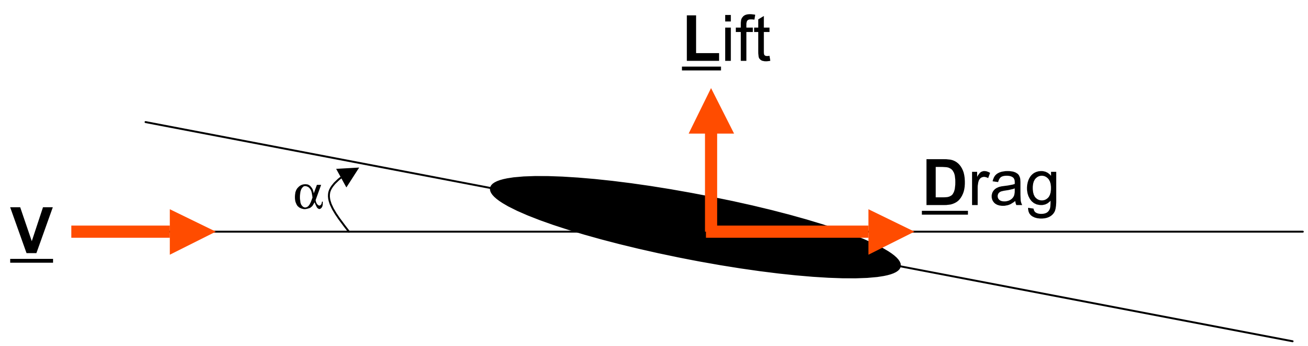


$$L = \frac{1}{2}\rho V^2 \cdot C_L \cdot S$$

$$D = \frac{1}{2}\rho V^2 \cdot C_D \cdot S$$

Dynamic Pressure (like in Bernoulli's equation!!)

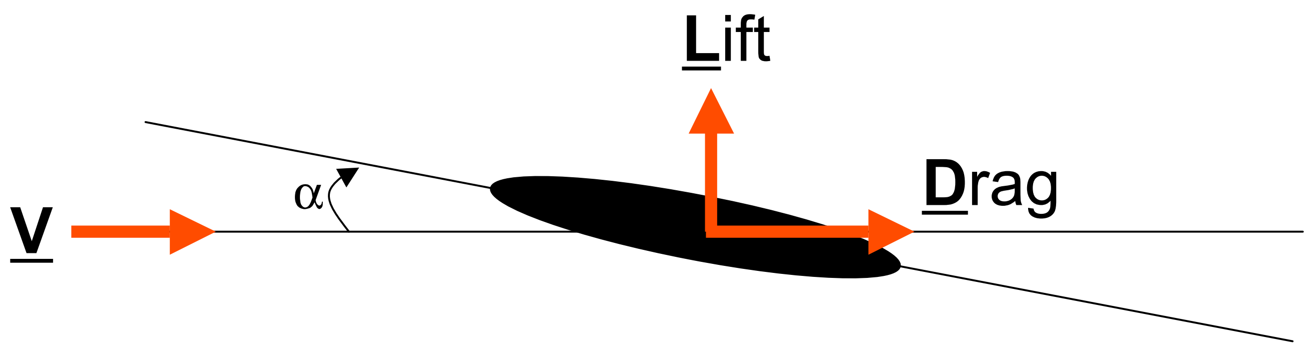
Aero/Hydro-dynamic Forces



$$L = \frac{1}{2}\rho V^2 \cdot \underline{\underline{C_L}} \cdot S \qquad D = \frac{1}{2}\rho V^2 \cdot \underline{\underline{C_D}} \cdot S$$

Empirical Force Coefficients

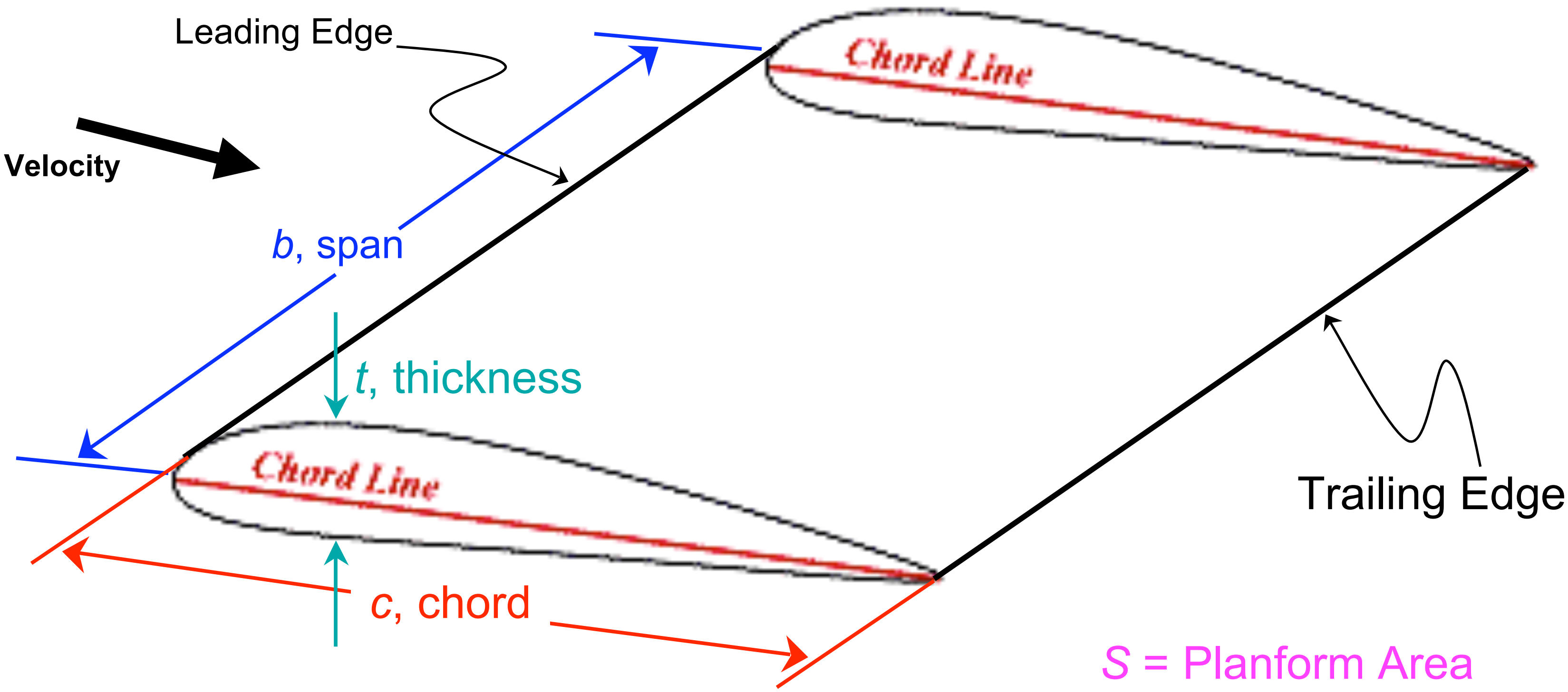
Aero/Hydro-dynamic Forces



$$L = \frac{1}{2}\rho V^2 \cdot C_L \cdot S \qquad D = \frac{1}{2}\rho V^2 \cdot C_D \cdot S$$

Wing Planform Area

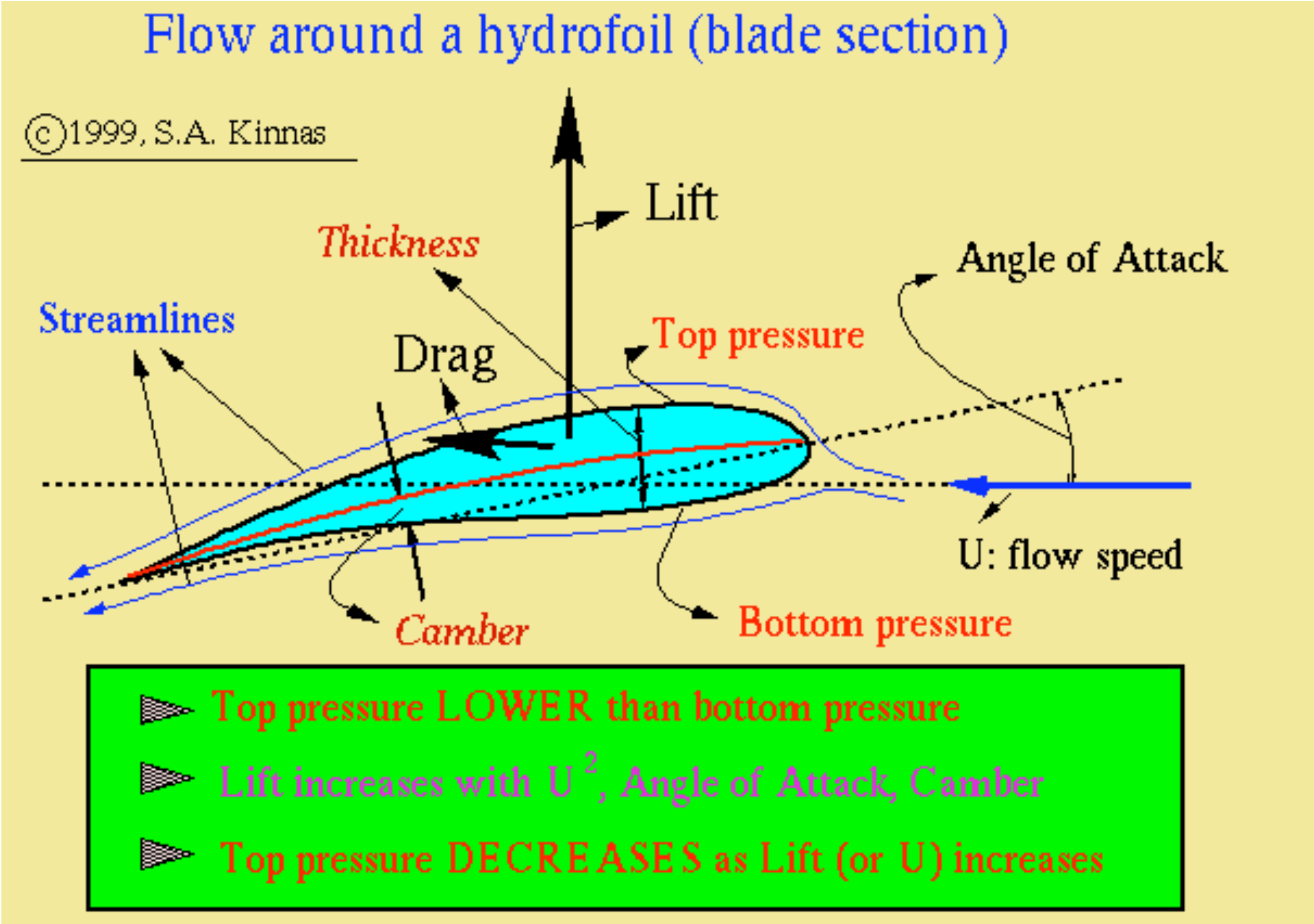
Aero/Hydro-foil Geometry



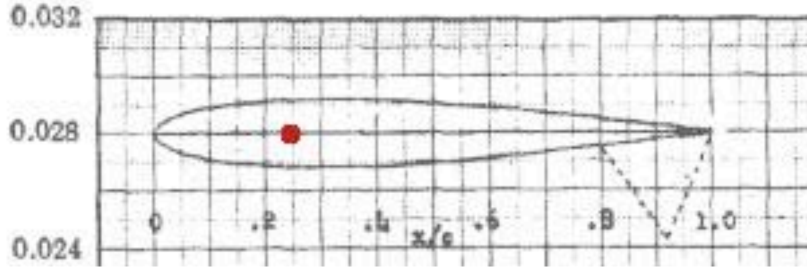
S = Planform Area
S = b*c for rectangular foil

Aspect Ratio: $AR = b^2 / S$

Lift on a hydrofoil



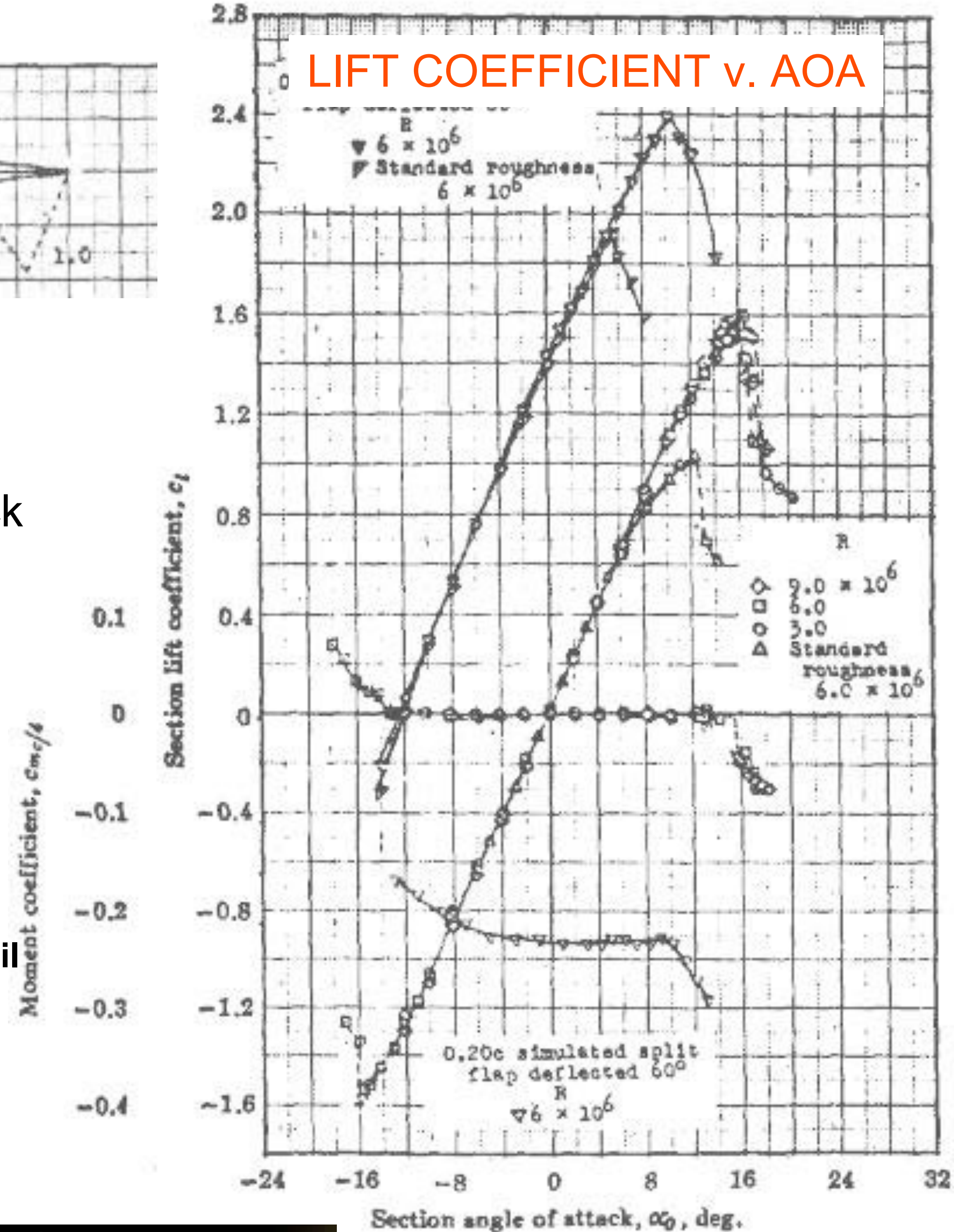
Courtesy of Spyros A. Kinnas. Used with permission.



Coefficient of Lift for a
NACA 0012 Airfoil as
A function of angle of attack

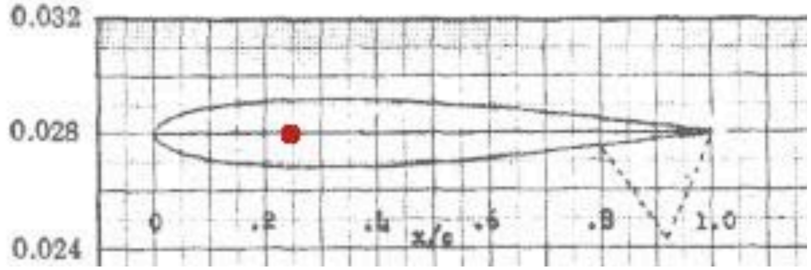
NACA 0012

First two numbers indicate camber
Double zeros indicate symmetric foil



National Advisory Committee for Aeronautics (NACA)

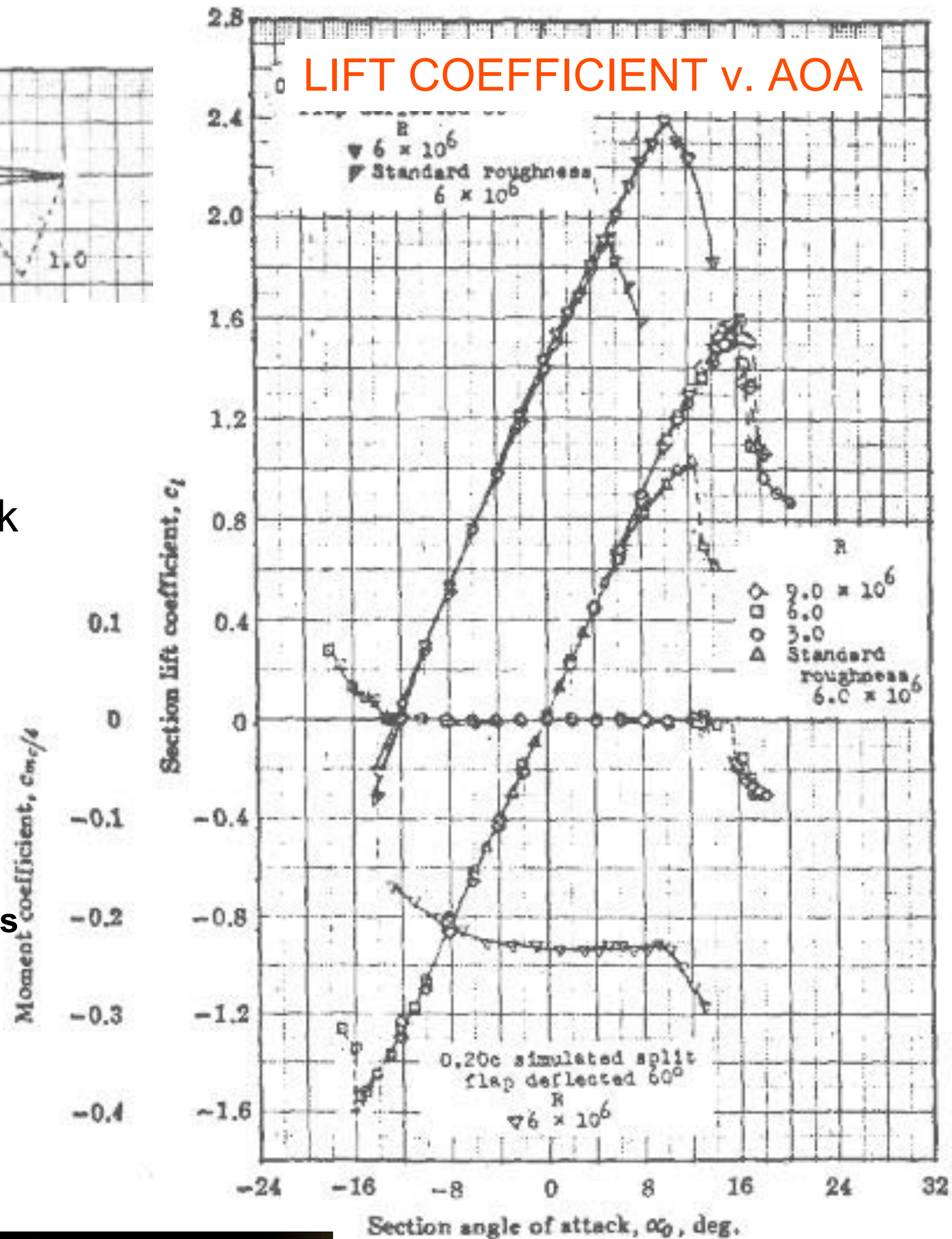
NACA 0012 Wing Section



Coefficient of Lift for a
NACA 0012 Airfoil as
A function of angle of attack

NACA 0012

Last two numbers indicate thickness of foil as % of total chord length



NACA 0012 Wing Section

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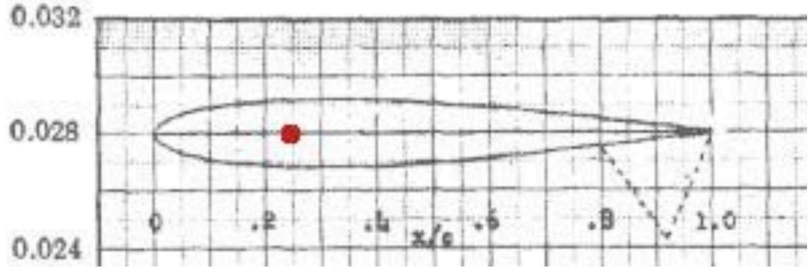


Coefficient of Lift for a
NACA 0012 Airfoil as
A function of angle of attack

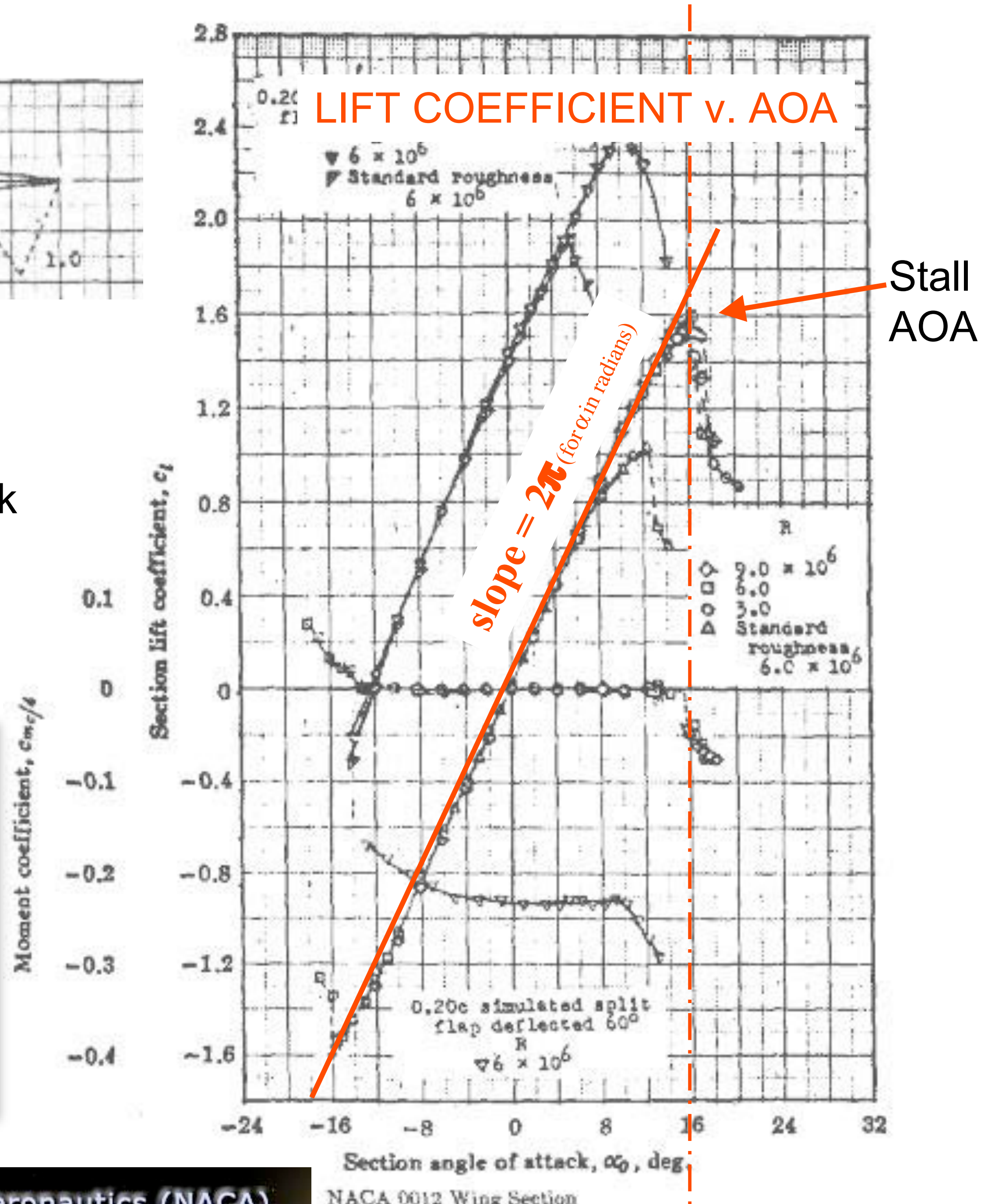
NACA 0012

For a symmetrical foil:

$$C_l = 2\pi\alpha$$

(α in radians!)

$$F_{LIFT} = \tfrac{1}{2}\rho U^2 \, C_l \, S$$

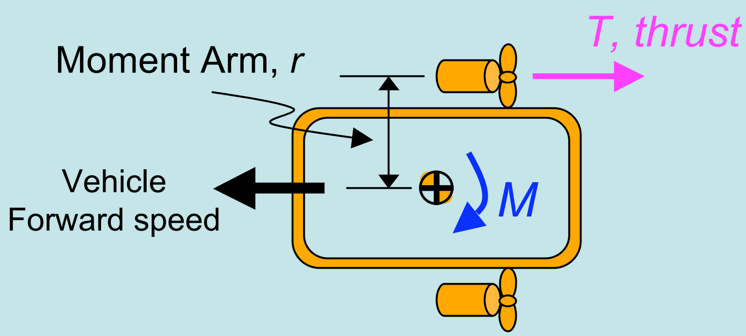


National Advisory Committee for Aeronautics (NACA)

Maneuvering with a Rudder

Turning Moment on a Vehicle

Single Motor Turn



M = r x T

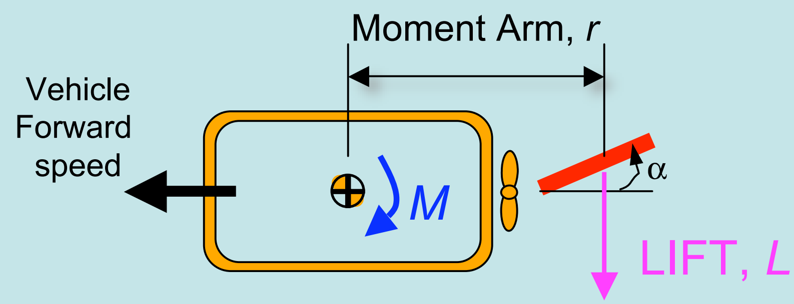
Sum of the Moments (Torques) about the CG equals the time rate of change of angular momentum

$$\Sigma M_{CG} = I\ddot{\theta}$$

*I = moment of Inertia of the vehicle about CG**

**can calculate in Solidworks!*

Turning a Vehicle with a Rudder



M = r x L

Lift on the Rudder:

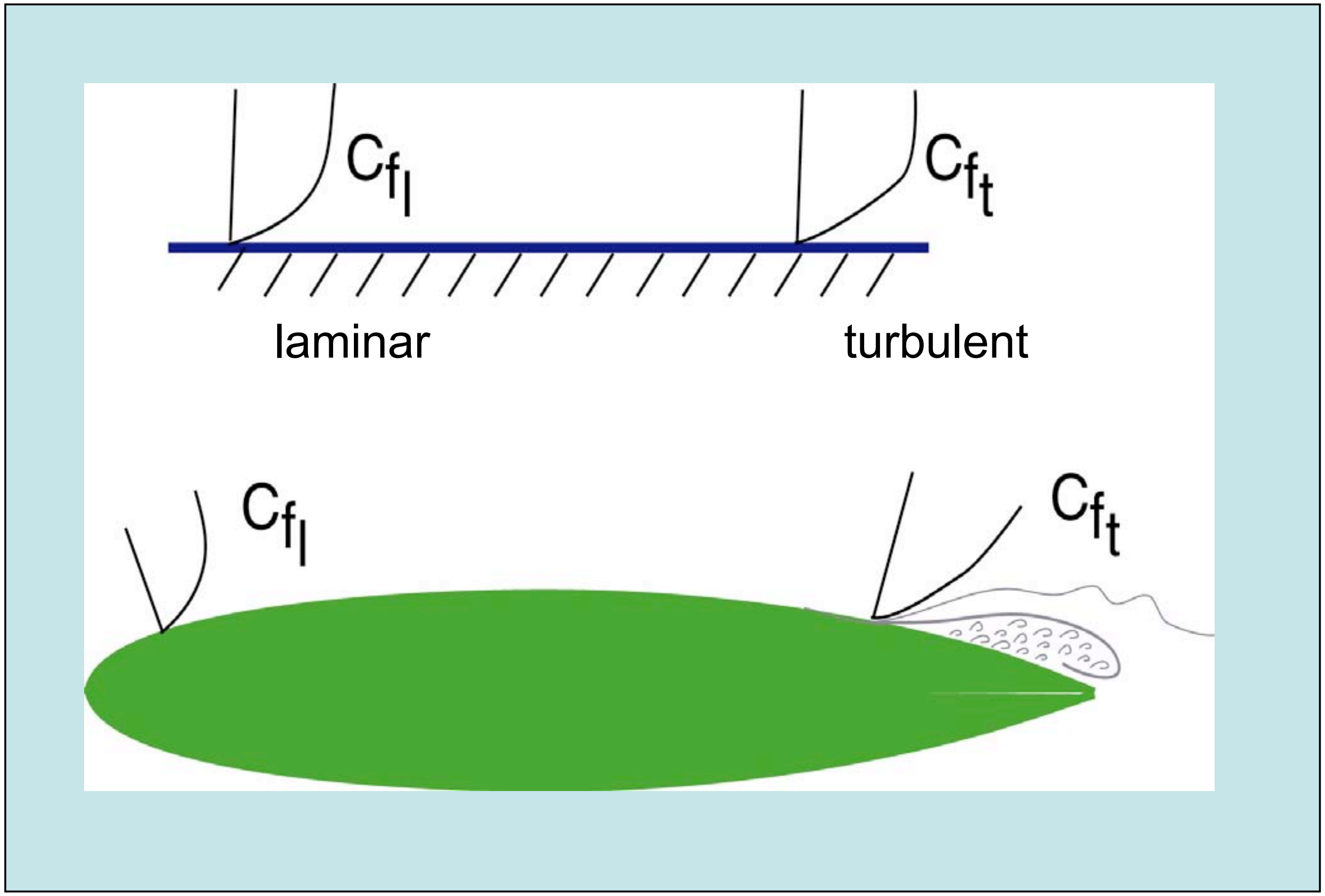
$$L = \tfrac{1}{2}\rho V^2 (2\pi\alpha) A$$

A = area of rudder
α in radians

Coefficient of Lift:

$$C_L = 2\pi\alpha$$

Viscous Drag



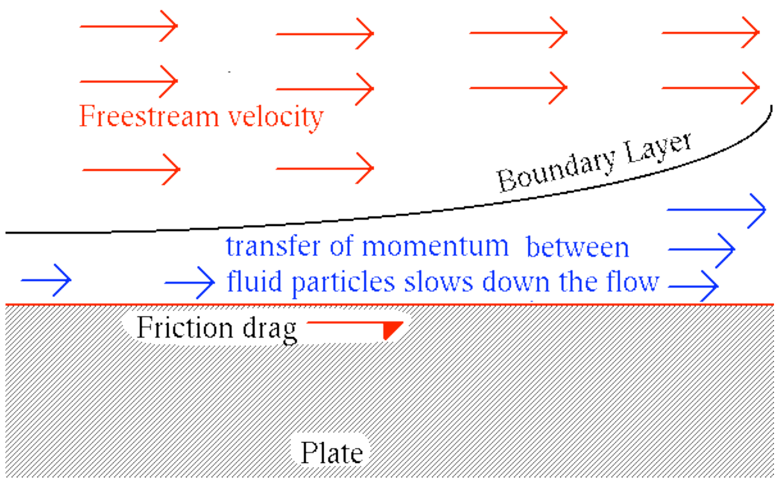
Skin Friction Drag: C_f

Form Drag: C_D due to pressure (turbulence, separation)

Streamlined bodies reduce separation, thus reduce form drag.
Bluff bodies have strong separation thus high form drag.

Friction Drag

The transfer of momentum between the fluid particles slows the flow down causing drag on the plate. This drag is referred to as friction drag.



<u>Friction Drag Coefficient:</u>

units

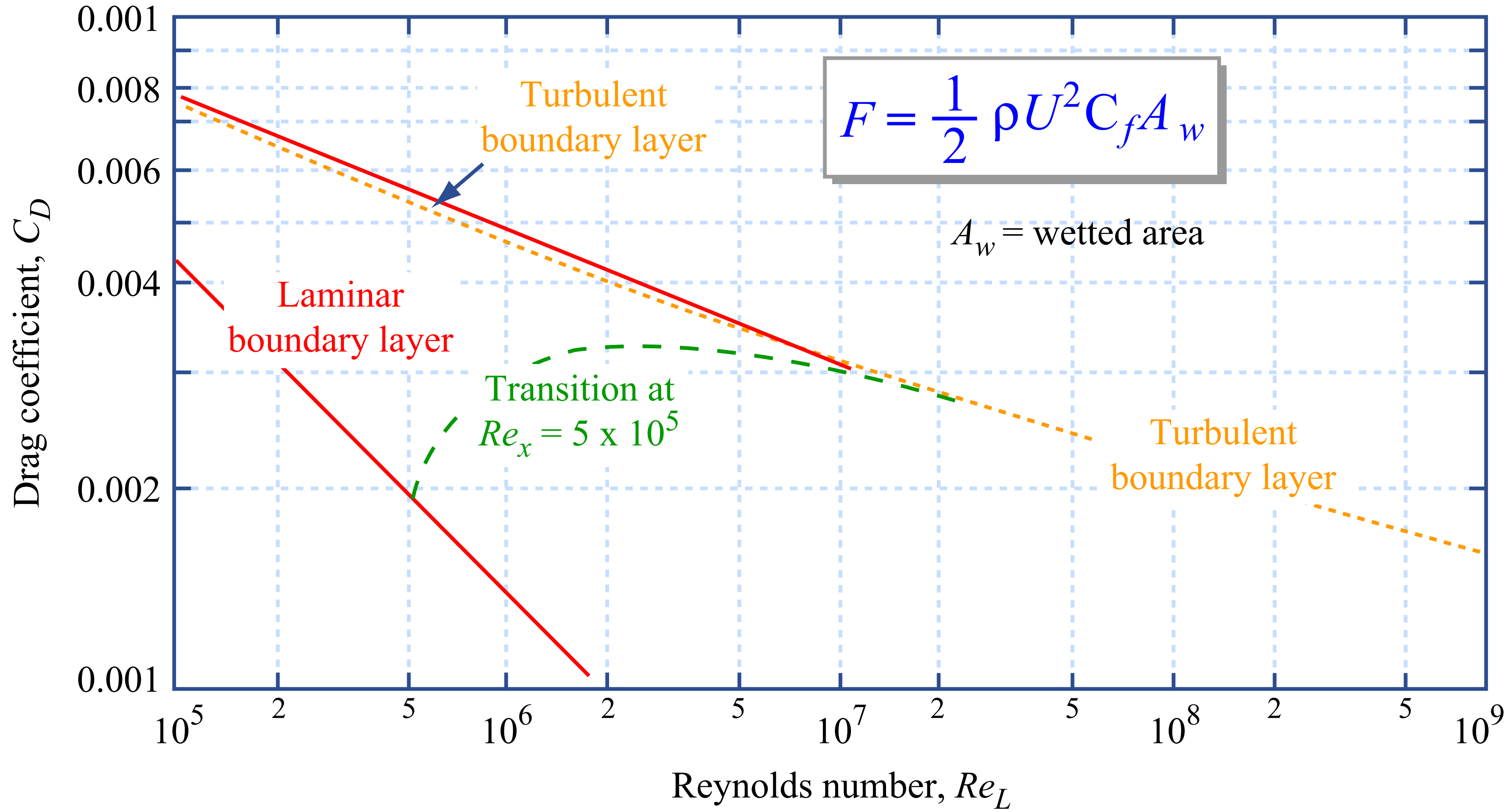
$$C_f = \frac{F}{\frac{1}{2}\rho U^2 A_w} \qquad \frac{[MLT^{-2}]}{[MLT^{-2}]}$$

(non-dimensional coefficient)

A_w = Wetted Area

THIS IS A SHEAR FORCE THAT COMES FROM SHEAR STRESS AT THE WALL!

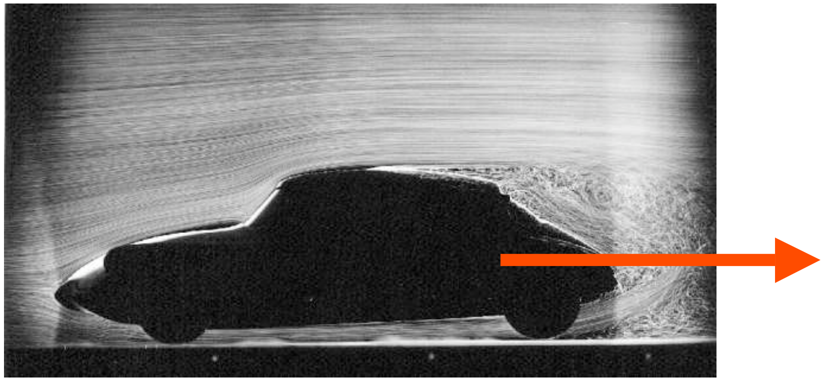
Flat Plate Friction Coefficient



Variation of drag coefficient with Reynolds number for a smooth flat plate parallel to the flow.

Viscous flow around bluff bodies (like cylinders) tends to separate and form drag dominates over friction drag.

Drag on Bodies



DRAG ACTS INLINE WITH VELOCITY

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Please see http://www.onera.fr/photos-en/tunnel/images/220006.jpg

Form Drag

- Drag Force on the body due to viscous effects:

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Please see http://www.onera.fr/photos-en/tunnel/images/220006.jpg

$$F_D = \frac{1}{2}\rho U^2 C_D A$$

- Where C_D is found empirically through experimentation
- A is profile (frontal) area
- C_D is Reynolds number dependent and is quite different in laminar vs. turbulent flows

Form Drag or Separation Drag or Pressure Drag (same thing!)

Flow Separating from a Cylinder

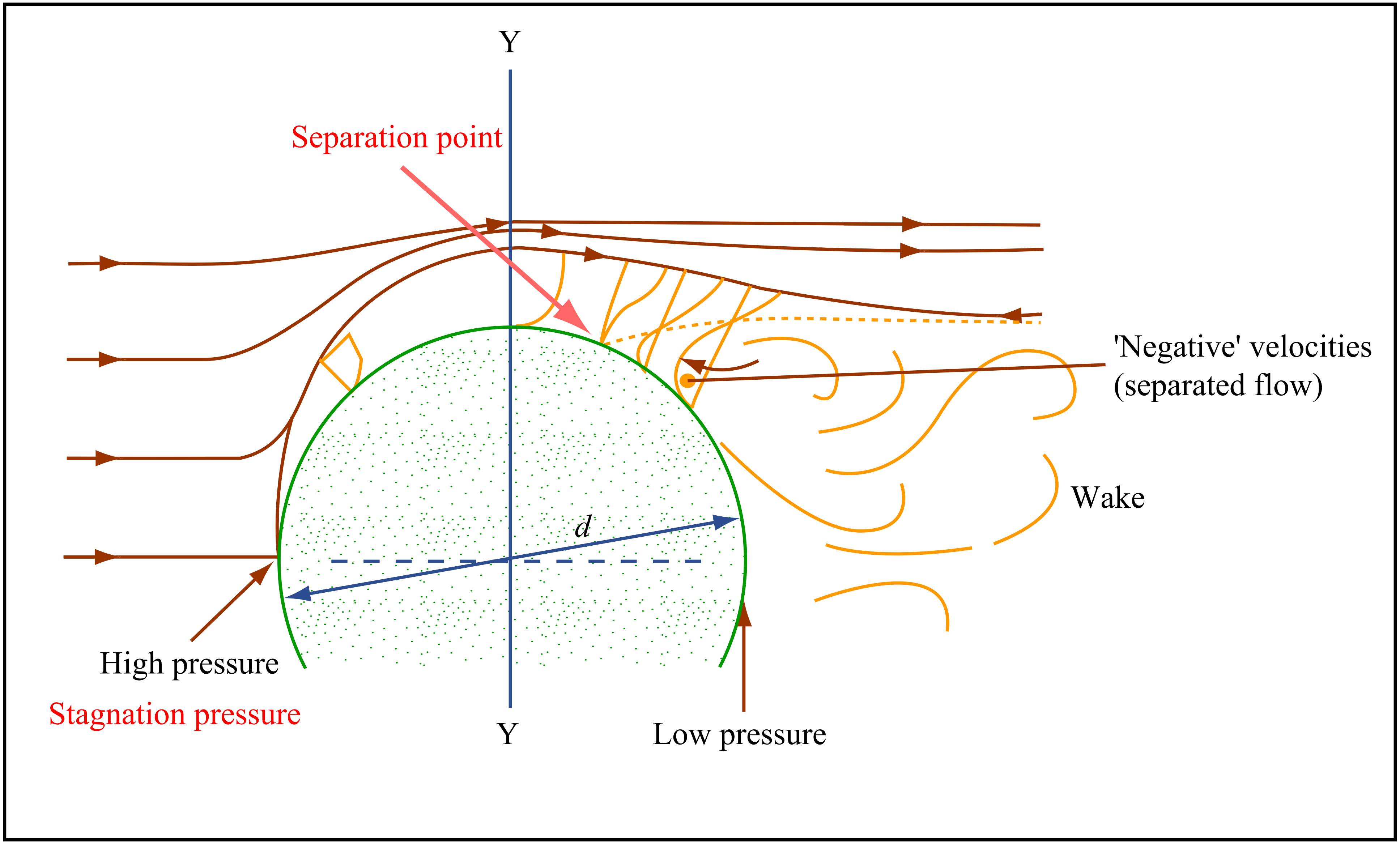

Figure by MIT OpenCourseWare.

Classical Vortex Shedding

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Please see http://en.wikipedia.org/wiki/File:Viv2.jpg

Alternately shed opposite signed vortices

Vortex shedding results from a wake instability

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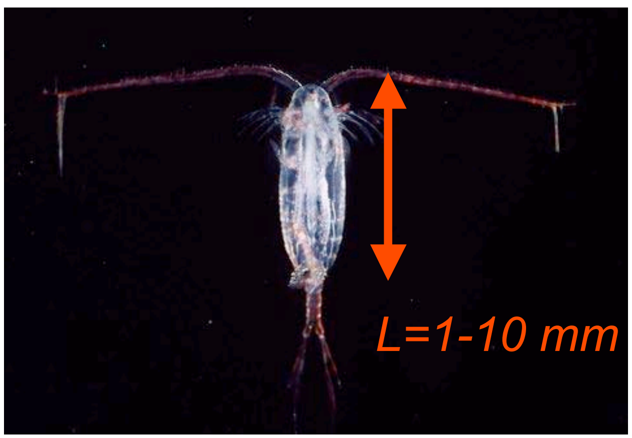
Please see Fig. 25-32 in Homann, Fritz. "Einfluß großer Zähigkeit bei Strömung um Zylinder."
Forschung auf dem Gebiete des Ingenieurwesens 7 (January/February 1936): 1-10.

Reynolds Number?

Non-dimensional parameter that gives us a sense of the ratio of inertial forces to viscous forces

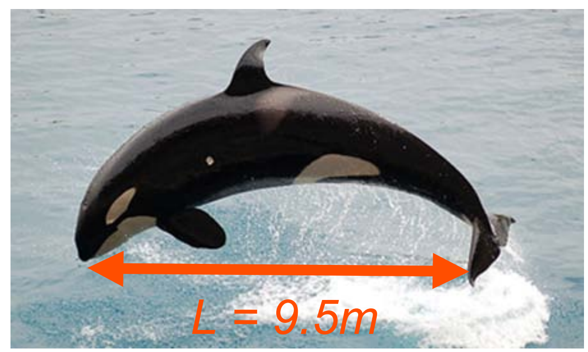
$$\mathrm{Re} = \frac{(\text{inertial forces})}{(\text{viscous forces})} = \frac{(\rho U^2 L^2)}{(\mu U L)} = \frac{\rho U L}{\mu}$$



Courtesy NOAA.

Speeds: around of 2*L/second

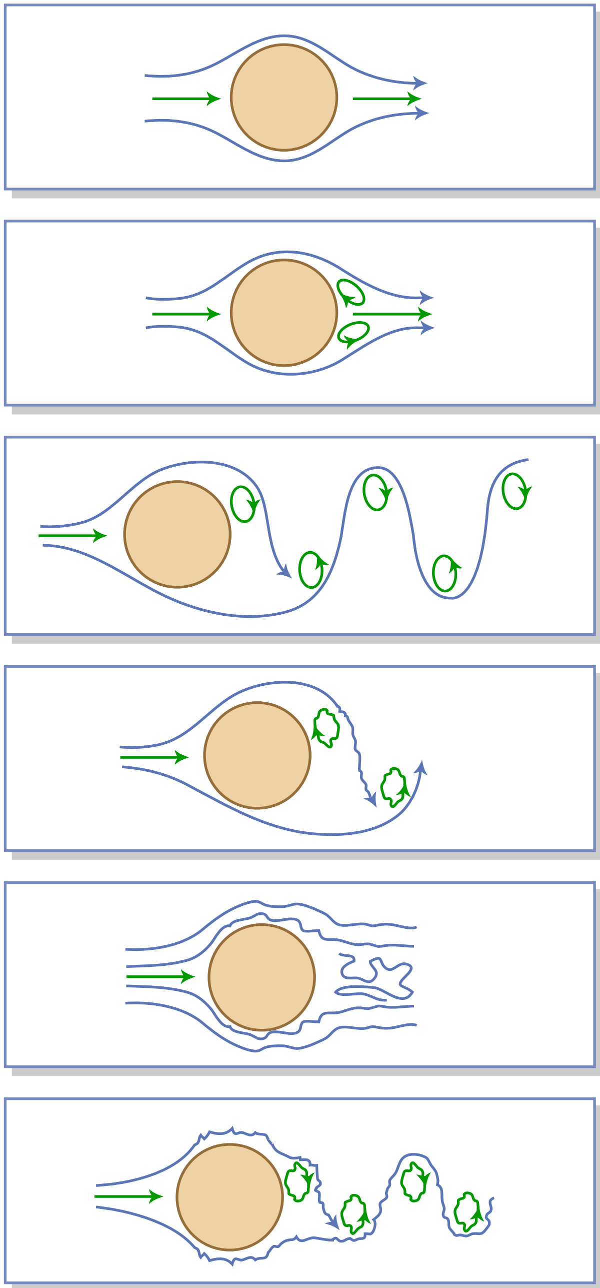
$$\nu_{water} = \rho/\mu = 10^{-6} m^2/s$$



Speeds: in excess of 56 km/hr

Image from Wikimedia Commons, http://commons.wikimedia.org

Reynolds Number Dependence



Regime of Unseparated Flow

A Fixed Pair of Foppl Vortices in Wake

Two Regimes in which Vortex Street is Laminar

Transition Range to Turbulence in Vortex

Vortex Street is Fully Turbulent

Laminar Boundary Layer has Undergone Turbulent Transition and Wake is Narrower and Disorganized

Re-establishment of Turbulent Vortex Street

Regimes of fluid flow across smooth circular cylinders (Lienhard, 1966).

From Blevins

$$Re = \frac{(\text{inertial forces})}{(\text{viscous forces})} = \frac{(\rho U^2 L^2)}{(\mu U L)} = \frac{\rho U L}{\mu}$$

$R_d < 5$

$5\text{-}15 < R_d < 40$

$40 < R_d < 150$

$150 < R_d < 300$

Transition to turbulence

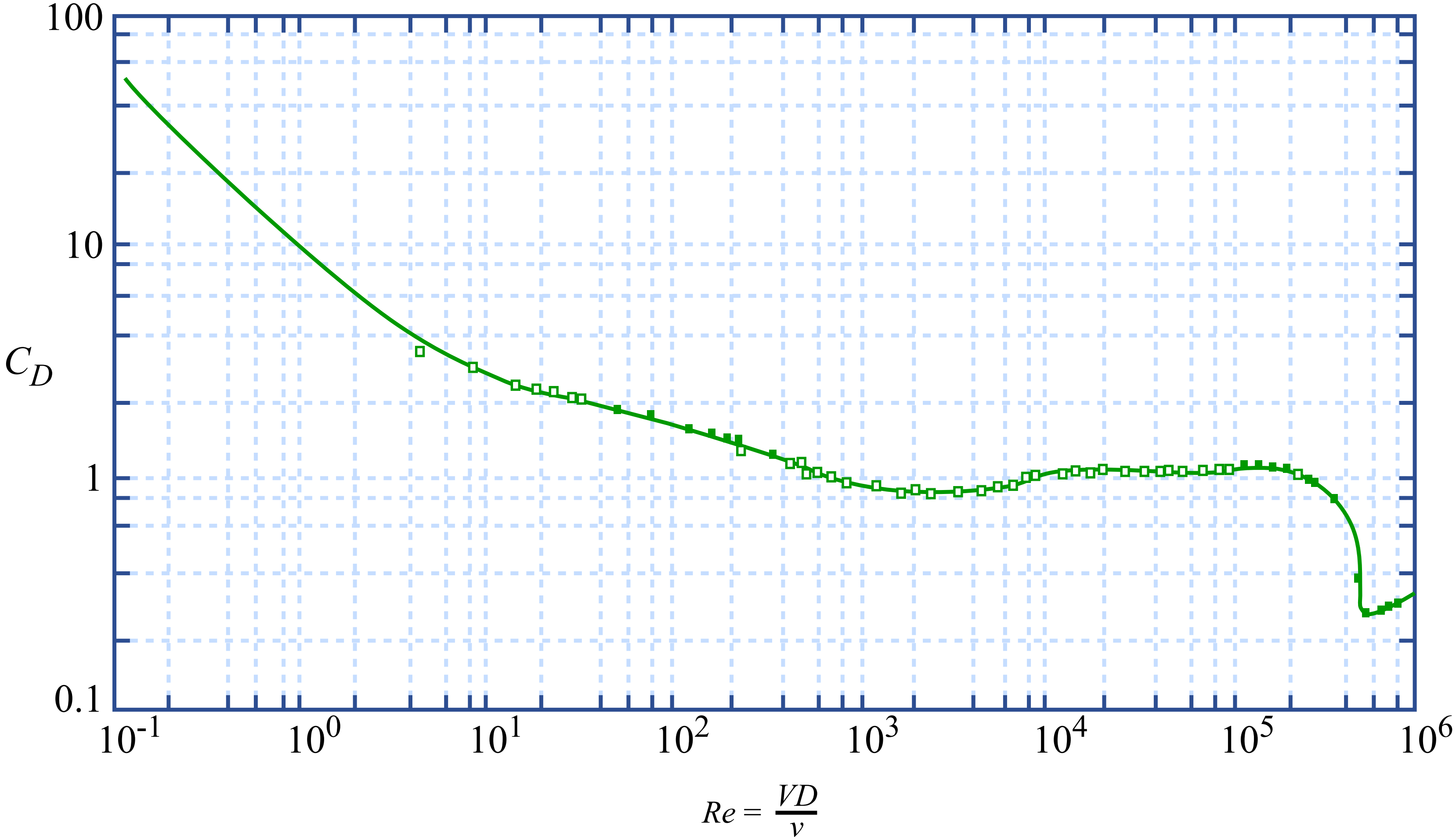
$300 < R_d < 3*10^5$

$3*10^5 < R_d < 3.5*10^6$

$3.5*10^6 < R_d$

Figure by MIT OpenCourseWare.

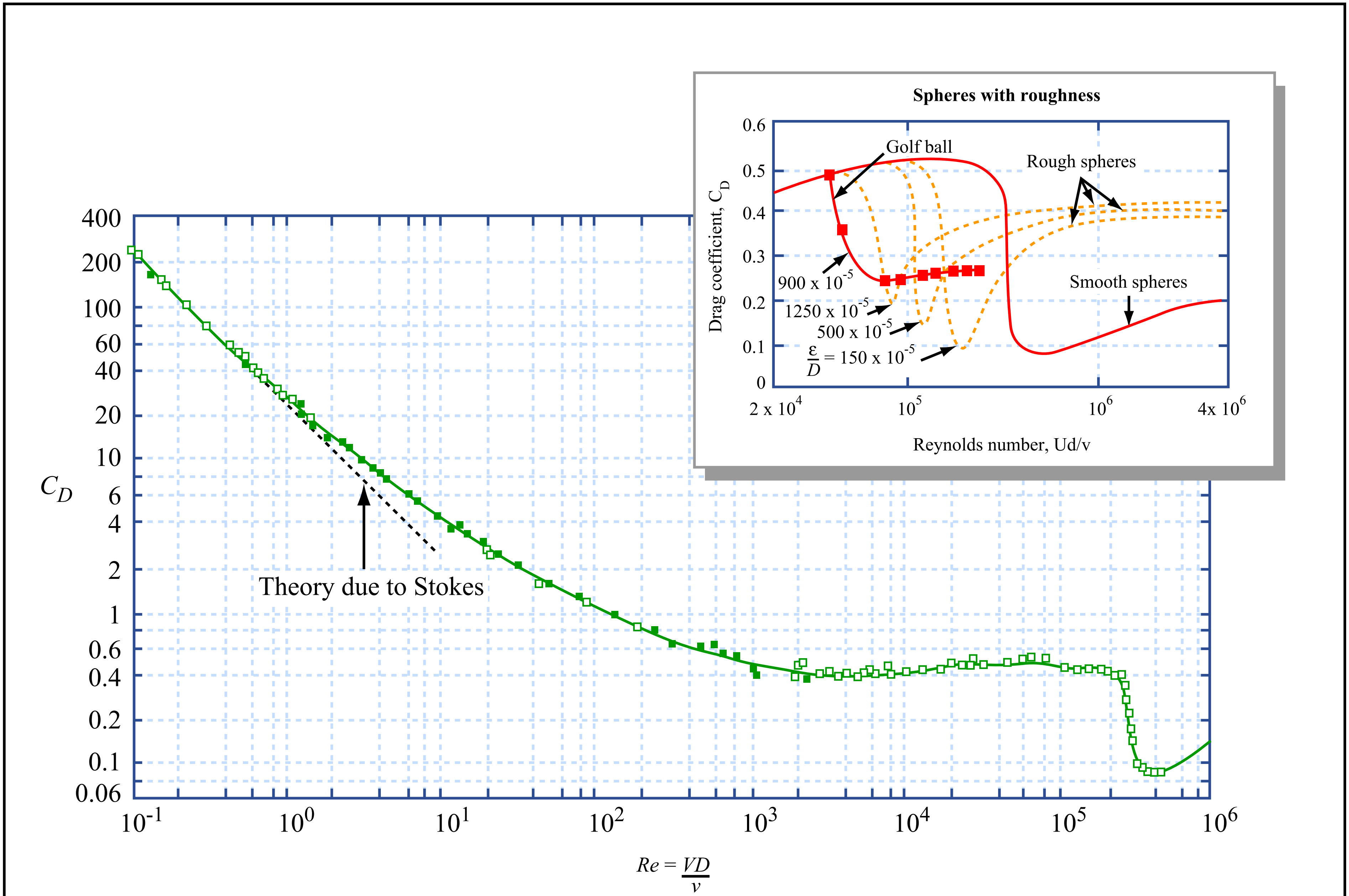
Drag Coefficient: Cylinder



Drag coefficient for a smooth circular cylinder as a function of Reynolds number.

Figure by MIT OpenCourseWare.

Drag Coefficient: Sphere



Drag coefficient of a smooth sphere as a function of Reynolds number.

Figure by MIT OpenCourseWare.

Trade-off between Friction and Pressure drag

c = Body length inline
with flow
t = Body thickness

2D Drag Coefficients

For 2D shapes: use C_D to calculate force per unit length.

Use a "strip theory" type approach to determine total drag, assuming that the flow is uniform along the span of the body.

Trade-off between Friction and Pressure drag

c = Body length inline with flow
t = Body thickness

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Please see Table 7.2 in White, Frank M. *Fluid Mechanics*. Boston, MA: McGraw-Hill, 2007.

3D Drag Coefficients

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Please see Table 7.3 in White, Frank M. *Fluid Mechanics*. Boston, MA: McGraw-Hill, 2007.

More 3D Shapes

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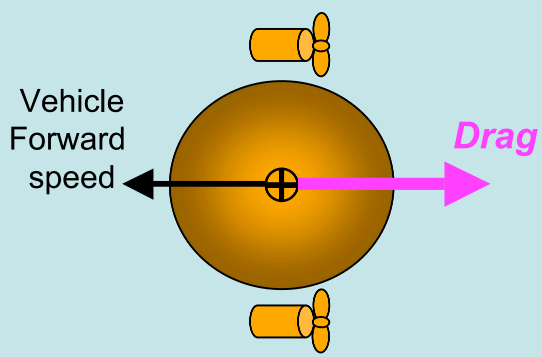
Calculating Drag on a Simple Structure

Example: Spherical vehicle with motors



$$Drag_{total} = Drag_{sphere} + 2 * Drag_{motors}$$

$$Drag_{sphere} = {}^{1}/_{2}\, \rho V^2\, C_D\, A$$

A = front area = πr^2

C_D = drag coefficient

(depends on velocity)

C_D = 0.4-0.5 (laminar)
or 0.2 (turbulent)

$$Drag_{motors} \sim {}^{1}/_{2}\, \rho V^2\, C_D\, A \;**$$

A = front area = πr^2

C_D = drag coefficient

(depends on aspect ratio)

Assuming L/D = 2

C_D=0.85

***This neglects the propellers which add some drag to the vehicle*

Use linear superposition to find total drag on a complex shape

Fluid Forces

Force on a surface ship is a function of X??

1) Fluid properties: density (ρ) & viscosity (μ)
2) Gravity (g)
3) Fluid (or body) velocity (U)
4) Body Geometry (L)

$$F = f(\rho, \mu, g, U, L)$$

Dimensional Analysis:

1) "Output" variable (Force) is a function of $N-1$ "input" variables (ρ, μ, g, U, L). Here $N=6$.
2) There are $M=3$ primary dimensions (units) for the variables listed above [Mass, Length, Time]
3) We can determine $P=N-M$ non-dimensional groups $(P=3)$.
4) How do we find these groups?

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Dimensional Analysis

$$F = f(\rho, \mu, g, U, L)$$

	F [kg m/s^2]	ρ [kg/m^3]	U [m/s]	L [m]	μ [kg/m/s]	g [m/s^2]
Mass [M]	1	1	0	0	1	0
Length [L]	1	-3	1	1	-1	1
Time [t]	-2	0	-1	0	-1	-2

Step 1) Make all variables containing mass non-dimensional in mass by dividing through by density: F/ρ and μ/ρ

Dimensional Analysis

$$\frac{F}{\rho} = f\left(\frac{\mu}{\rho}, g, U, L\right)$$

	F/ρ [m^4/s^2]	ρ/ρ [--]	U [m/s]	L [m]	μ/ρ [m^2/s]	g [m/s^2]
Mass [M]	0	0	0	0	0	0
Length [L]	4	0	1	1	2	1
Time [t]	-2	0	-1	0	-1	-2

Step 2) Rewrite Matrix deleting density column and mass row

Dimensional Analysis

$$\frac{F}{\rho} = f\left(\frac{\mu}{\rho}, g, U, L\right)$$

	F/ρ [m^4/s^2]	U [m/s]	L [m]	μ/ρ [m^2/s]	g [m/s^2]
Length [L]	4	1	1	2	1
Time [t]	-2	-1	0	-1	-2

Step 3) Non-dimensionalize all variables containing time, using velocity: $F/\rho U^2$ and $\mu/\rho U$ and g/U^2

Dimensional Analysis

$$\frac{F}{\rho U^2} = f\left(\frac{\mu}{\rho U}, \frac{g}{U^2}, L\right)$$

	$F/\rho U^2$ [m^2/s^0]	U/U [--]	L [m]	$\mu/\rho U$ [m^1/s^0]	g/U^2 [m^{-1}/s^0]
Length [L]	2	0	1	1	-1
Time [t]	0	0	0	0	0

Step 4) Rewrite Matrix

Dimensional Analysis

$$\frac{F}{\rho U^2} = f\left(\frac{\mu}{\rho U}, \frac{g}{U^2}, L\right)$$

	$F/\rho U^2$ [m^2/s^0]	L [m]	$\mu/\rho U$ [m^1/s^0]	g/U^2 [m^{-1}/s^0]
Length [L]	2	1	1	-1

Step 5) Non-dimensionalize all variables containing length: $F/\rho U^2 L^2$ and $\mu/\rho UL$ and gL/U^2

Dimensional Analysis

$$\frac{F}{\rho U^2 L^2} = f\left(\frac{\mu}{\rho U L}, \frac{gL}{U^2}\right)$$

	$F/\rho U^2 L^2$ [m^0]	L/L [--]	$\mu/\rho UL$ [m^0]	gL/U^2 [m^0]
Length [L]	0	0	0	0

Step 5) Your equation is non-dimensional! Yea!

But does it make sense??

Classical non-dimensional parameters in fluids

$$\frac{F}{\rho U^2 L^2} = f\left(\frac{\mu}{\rho U L}, \frac{gL}{U^2}\right)$$

$$C_F = \frac{F}{\frac{1}{2}\rho U^2 L^2}$$

Force Coefficient

(can be found through experiments and is considered an "empirical" coefficient, L^2 is equivalent to Area of object)

$$\mathrm{Re} = \frac{\rho U L}{\mu}$$

Reynolds Number

(important for all forces in air or water)

$$Fr = \frac{U^2}{gL}$$

Froude Number

(in fluids typically only important when near surface of ocean/water)