2.003J/1.053J Dynamics and Control I, Spring 2007
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Recitation 1

Kinematics

Rigid Body Dynamics - 2.003J
Effects of Newton's Laws on particles
Particles attached rigidly (constrained to move together)
2.005, 2.006: Flow
Continuum Mechanics (Heat: Vibration of particles)
2.001
Connected but not rigid
Change distance elatically

Kinematics underlies dynamics.

Newton's Laws

$$F = ma \rightarrow \text{ Single Particles} \rightarrow \text{ Multiple Particles}$$

action = reaction

Kinematics
No gravity. Newton's Laws do not apply.
Related to geometry (in cartoonland). Geometry without Newton.

Standing on floor. Do not move because floor stops you. Still have gravity though. Kinematic constraint.

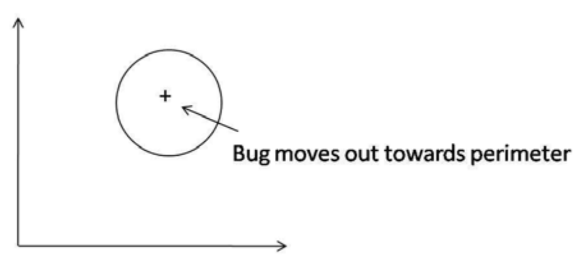

Figure 1: Standing in a corner, one throws a frisbee. The bug crawls along the frisbee. Figure by MIT OCW.

What is its velocity and acceleration?

What do you need?
Need to know how fast frisbee is moving (Initial Velocity and Angular Velocity). Air drag is dynamics (force).
Why do we care?
Spaceship spinning, revolving around earth, astronaut stuck by magnetic boots
Need to know center of mass movement in time.

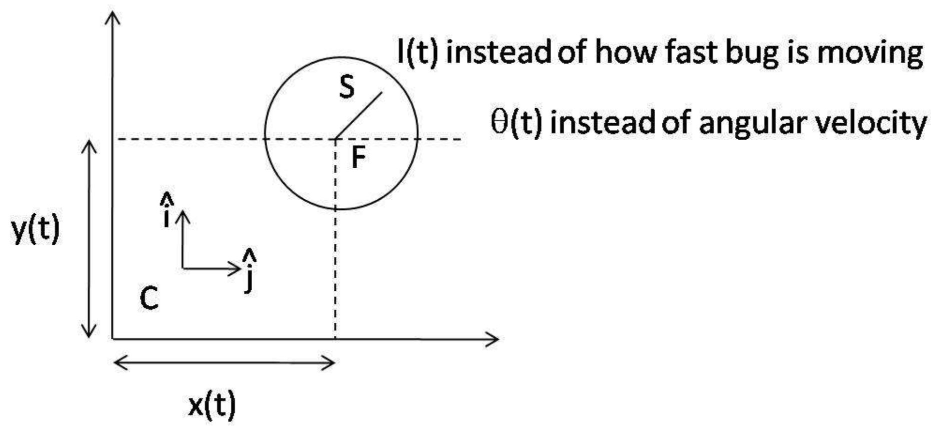

Figure 2: $x(t)$ and $y(t)$ describe the frisbee position as a function of time. Figure by MIT OCW.

$x(t), y(t), l(t), \theta(t)$ What is the velocity? Acceleration?

Find position function, then differentiate.
Use vectors.

$$\underline{r}^{CF} = \hat{\imath}x(t) + \hat{\jmath}y(t) \text{ Corner to Frisbee}$$
$$\underline{r}^{FS} = \hat{\imath}l\cos\theta + \hat{\jmath}\sin\theta \text{ (Stop writing (t))}$$

Suppose $x(t) = y(t) = 0$. $\underline{r}^{CF} = \underline{r}^{FS}$. Frisbee spinning on figure. Suppose $x(t), y(t) \neq 0$ then $\frac{d\underline{r}^{FS}}{dt}$ would give velocity of bug as $v \neq$ were hovering over frisbee as it moves. $\underline{r}^{CF}$ is position of frisbee center of mass. $\frac{d}{dt}\hat{\imath} = 0$. $\hat{\imath}$ is stationary because room is stationary. (Frame is stationary).

$$\underline{r}^{CS} = \hat{\imath}[x + l\cos\theta] + \hat{\jmath}[y + l\sin\theta]$$

Derivative of which equation? Depends on frame of reference.

$$\underline{v}^s = \hat{\imath}[\dot{x} + \dot{l}\cos\theta - l\dot{\theta}\sin\theta] + \hat{\jmath}[\ldots]$$

$$\underline{a}^s = \frac{d\underline{v}^s}{dt} = \hat{\imath}[\ddot{x} + \ddot{l}\cos\theta - \dot{\theta}\dot{l}\sin\theta - \dot{l}\dot{\theta}\sin\theta - l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta] + \hat{\jmath}[\ddot{y} + \ddot{l}\sin\theta + 2\dot{l}\dot{\theta}\cos\theta + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta]$$

$$\underline{a}^s = \hat{\imath}[\ddot{x}+\ddot{l}\cos\theta - 2\dot{l}\dot{\theta}\sin\theta - l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta] + \hat{\jmath}[\ddot{y}+\ddot{l}\sin\theta + 2\dot{l}\dot{\theta}\cos\theta + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta]$$

$\ddot{x}$: Linear acceleration of frisbee

$\ddot{x} + \ddot{l}\cos\theta$: Linear acceleration of bug assuming θ is constant

$2\dot{l}\dot{\theta}\sin\theta$: Coriolis. Earth is spinning. If run fast, inexplicable drift to right. "Fictitious force." Hurricanes only spin 1 way in northern hemisphere.

$l\ddot{\theta}\sin\theta$: Linear acceleration if frisbee is not flung but frisbee is rotating. Euler torque acceleration.

$l\dot{\theta}^2\cos\theta$: Centripetal Acceleration

Just found acceleration and velocity of this frisbee and bug.
Physicists took 200 years to solve this, because they would leave out terms.

Naval Battles:
- Gunners noticed guns would be off.
- Got repointed when gunners crossed equator 250 - 300 years ago. Discovery of coriolis effect.

$$F = m_b \underline{a}^s$$

Kinematics runs parallel to $F = ma$.

$${}^c\underline{a}^s = {}^c\frac{d\underline{v}^s}{dt}$$

$${}^c\underline{v}^s = {}^c\frac{d}{dt}\underline{r}^{cs}$$

Notation above specifies frame of reference C.
Derivative of vector → cross product of angular velocity.

$$\dot{\theta} = \frac{d\theta}{dt} = \text{ angular speed}$$

$$\dot{\theta}k \times \text{ something } \Rightarrow \text{ derivative}$$