7.2.1 Boolean Classification

A related problem is that of Boolean Classification [AABS15]. Let us restrict our attention to In error-correcting codes one wants to build a linear codebook that does not contain a codeword with weight $\leq d - 1$. In other words, one wants a linear codebook C that does intersect $B(d-1) = \{x \in \{0,1\}^n : 0 < \Delta(x) \leq d-1\}$ the pinched Hamming ball of radius d (recall that $\Delta(d)$ is the Hamming weight of x, meaning the number of non-zero entries). In the Boolean Classification problem one is willing to confuse two codewords as long as they are sufficiently close (as this is likely to mean they are

in the same group, and so they are the same from the point of view of classification). The objective then becomes understanding what is the largest possible rate of a codebook that avoids an Annulus $A(a,b) = \{x \in \{0,1\}^n : a \leq \Delta(x) \leq b\}$. We refer the reader to [AABS15] for more details. Let us call that rate

$$R_A^*(a, b, n).$$

Note that $R_A^*(1, d-1, n)$ corresponds to the optimal rate for a binary error-correcting code, conjectured to be $1 - H_q\left(\frac{d}{N}\right)$ (The GV bound).

Open Problem 7.2 *It is conjectured in [AABS15] (Conjecture 3 in [AABS15]) that the optimal rate in this case is given by*

$$R_A^*(\alpha n, \beta n, n) = \alpha + (1 - \alpha) R_A^* (1, \beta n, (1 - \alpha)) + o(1),$$

where $o(1)$ goes to zero as n goes to infinity.
    This is established in [AABS15] for $\beta \geq 2\alpha$ but open in general.

Tghgtgpeg

[AABS15]   E. Abbe, N. Alon, A. S. Bandeira, and C. Sandon. Linear boolean classification, coding and "the critical problem". *Available online at arXiv:1401.6528v3 [cs.IT]*, 2015.

18.S096 Topics in Mathematics of Data Science
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