6.6 The Kadison-Singer problem

The Kadison-Singer problem (or the related Weaver's conjecture) was one of the main questions in frame theory, it was solved (with a non-constructive proof) in the recent breakthrough of Marcus, Spielman, and Srivastava [MSS15b], using similar techniques to their earlier work [MSS15a]. Their theorem guarantees the existence of universal constants $\eta \geq 2$ and $\theta > 0$ s.t. for any tight frame $\omega_1, \ldots, \omega_N \in \mathbb{C}^M$ satisfying $\|\omega_k\| \leq 1$ and

$$\sum_{k=1}^{N} \omega_k \omega_k^T = \eta I,$$

there exists a partition of this tight frame $S_1, S_2 \subset [N]$ such that each is "almost a tight frame" in the sense that,

$$\sum_{k \in S_j} \omega_k \omega_k^T \preceq (\eta - \theta) I.$$

However, a constructive prove is still not known and there is no known (polynomial time) method that is known to construct such partitions.

Open Problem 6.5 *Give a (polynomial time) construction of the tight frame partition satisfying the properties required in the Kadison-Singer problem (or the related Weaver's conjecture).*

References

[MSS15a]   A. Marcus, D. A. Spielman, and N. Srivastava. Interlacing families i: Bipartite ramanujan graphs of all degrees. *Annals of Mathematics*, 2015.

[MSS15b]   A. Marcus, D. A. Spielman, and N. Srivastava. Interlacing families ii: Mixed characteristic polynomials and the kadison-singer problem. *Annals of Mathematics*, 2015.

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