Defn: $G_A^0 := \bigcap \{H : H \text{ is type-definable over } A,\ [G:H] < \infty\}$

is the A-connected component of G.

Want to prove (1) $[G : G_A^0] < \infty$ and (2) $G \trianglerighteq G_A^0$.

Proof (1) Since we only need to consider boundedly many H's.

(2) Since G_A^0 has bounded index, it has boundedly many conjugacy classes.

If $gG_A^0 = hG_A^0$ then $gG_A^0g^{-1} = hG_A^0h^{-1}$.

Let $H' = \bigcap \{\text{all conjugates of } G_A^0\}$. Then $[G : H'] < \infty$ and H' is A-invariant. $\Rightarrow$ H' is type definable$/A$ $\Rightarrow$ $G_A^0 \leq H'$. $\Rightarrow$ $G \trianglerighteq G_A^0$.

Exercise (Similar): $G_A^0 = G^0_{\mathrm{bdd}(A)}$.