18.703 HOMEWORK #6, DUE TUESDAY APRIL 3RD

1. Find a presentation of the dihedral group D_n.

2. Herstein, Chapter 2, §9, 2.

3. Herstein, Chapter 3, §3, 1.

4. Herstein, Chapter 3, §3, 2.

5. Herstein, Chapter 3, §3, 3.

6. Herstein, Chapter 3, §3, 6: Show that the 3-cycles generate A_n.

7. Herstein, Chapter 3, §3, 7.

8. Let P be a Sylow p-subgroup of a finite group G. Show that P is characteristically normal in G if and only if there is only one Sylow p-subgroup.

9. Let G be an abelian group of order $p^l q^m$, where p and q are distinct primes and $l > 0$, $m > 0$. Show that $G \simeq P \times Q$, where P is a subgroup of order p^l and where Q is a subgroup of order q^m.

10. Let G be a simple group of order 168. Show that G is isomorphic to a subgroup of S_8.

11. Show that every simple group whose order lies between 1 and 59 has prime order.

12. **Challenge Problem:** Herstein, Chapter 3, §3, 9 & 10:
(i) If $n \geq 5$ and $\{ e \} \neq N \triangleleft A_n$ is a normal subgroup, show that N must contain a 3-cycle.
(ii) Show that A_n is simple if and only if $n \neq 4$.

13. **Challenge Problem:** Show that every simple group whose order lies between 61 and 167 has prime order.

18.703 Modern Algebra

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