18.443 Problem Set 10 Spring 2015
Statistics for Applications
Due Date: 5/15/2015
prior to 3:00pm

Problems from John A. Rice, Third Edition. [$Chapter.Section.Problem$]

1. 12.5.1

   Solution: See R script/html $Problem\_12\_5\_1.r/html$

2. 12.5.6

   Prove this version of the Bonferroni inequality:

   $$P\left(\cap_{i=1}^{n} A_i\right) \geq 1 - \sum_{i=1}^{n} P(A_i^c)$$

   Let $A_* = \cap_{i=1}^{n} A_i$. Then

   $$A_*^c = \cup_{k=1}^{n} A_i^c$$

   So $P(A_*^c) = P(\cup_{k=1}^{n} A_i^c) \leq \sum_{k=1}^{n} P(A_i^c)$

   Because $P(A_*) = 1 - P(A_*^c)$, it follows that

   $$P(A_*) \geq 1 - \sum_{k=1}^{n} P(A_i^c)$$

   In the context of simulataneous confidence intervals

   A_i is the the event that the ith confidence interval contains the associated parameter

   $\cap_{i=1}^{n} A_i$ is the the event that all confidence intervals contains the associated parameters simultaneously.

3. 12.5.17. Find the mle's of the parameters $\alpha_i, \beta_j, \delta_{ij}$, and μ of the model for the two-way layout.

   The model for the two-way layout is given by:

   $$Y_{ijk} = \mu + \alpha_i + \beta_j + \delta_{ij} + \epsilon_{ijk}$$

   where

   $$\sum_{i=1}^{I} \alpha_i = 0$$
   $$\sum_{j=1}^{J} \beta_j = 0$$
   $$\sum_{i=1}^{I} \delta_{ij} = \sum_{j=1}^{J} \delta_{ij} = 0$$

   As noted in Rice (p. 493) the log likelihood is

$l = -\frac{IJK}{2} log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} (Y_{ijk} - \mu - \alpha_i - \beta_j - \delta_{ij})^2$

Solving the following equations:

$$\frac{\partial l}{\partial \mu_i} = 0 \Longrightarrow \hat{\mu} = \overline{Y_{...}}$$

$$\frac{\partial l}{\partial \alpha_i} = 0 \Longrightarrow \hat{\alpha}_i = \overline{Y_{i..}} - \hat{\mu}$$

$$\frac{\partial l}{\partial \beta_i} = 0 \Longrightarrow \hat{\beta}_i = \overline{Y_{.j.}} - \hat{\mu}$$

$$\frac{\partial l}{\partial \delta_{ij}} = 0 \Longrightarrow \hat{\delta}_{ij} = \overline{Y_{ij.}} - \hat{\mu} - \hat{\alpha}_i - \hat{\beta}_j$$

These terms yield the answer formulas of the problem.

4. 13.8.1. See R script html $Problem\_13\_8\_1.html$

5. 13.8.25. See R script html $Problem\_13\_8\_25.html$

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