



**cache hit:** CPU needs item in cache (fast)

**cache miss:** CPU needs item not in cache

— item loaded into cache for future use, replacing some other item

**optimal replacement:** on cache miss, loaded item replaces item that will not be needed for the *longest time in the future*

[ more realistic scheme: **LRU replacement** — replace least recently used item  
— provably within small constant factor of optimal, but much harder to analyze ]

**fully associative** — any item in memory can go anywhere in the cache

[ real caches have limited associativity, which causes “unlucky”

memory-access patterns to go same place in cache

...effectively shrinks cache in these cases ]

**temporal locality** — same item is re-used for several computations that are close to one another in time  $\Rightarrow$  still in-cache  $\Rightarrow$  efficient

[ there is also **spatial locality** — items close to one another in main memory are used close in time ... exploited by **cache lines**, TBD ]

**cache complexity** — the number of cache misses  $Q(n; Z)$  required for a given algorithm running on a problem of size  $n$  with cache of size  $Z$

... usually only given as **asymptotic** result for large  $n, Z$ , ignoring constant factors

**asymptotic notation:**

we say a function  $f(n)$  is  $O(g(n))$  if  $g(n)$  is an **asymptotic upper bound** for  $f(n)$ , ignoring constant factors. Technically, if  $|f(n)| < C |g(n)|$  for some constant  $C > 0$  for all sufficiently large  $n$  (i.e., for all  $n > N$  for some  $N$ )

we say a function  $f(n)$  is  $\Omega(g(n))$  if  $g(n)$  is an **asymptotic lower bound** for  $f(n)$ , ignoring constant factors. Technically, if  $|f(n)| > C |g(n)|$  for some constant  $C > 0$  for all sufficiently large  $n$  (i.e., for all  $n > N$  for some  $N$ )

we say a function  $f(n)$  is  $\Theta(g(n))$  if  $g(n)$  is an **asymptotic tight bound** for  $f(n)$ , ignoring constant factors. Technically, if  $f(n)$  is *both*  $O(g(n))$  and  $\Omega(g(n))$

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