18.335 Problem Set 4

Problem 1: Almost GMRES

We use the Arnoldi method to build up an orthogonal basis Q_n for $\mathscr{K}_n$, with $AQ_n = Q_n H_n + h_{n+1,n} q_{n+1} e_n^* = Q_{n+1}\tilde{H}_n$. GMRES then finds an approximate solution to $Ax = b$ by minimizing $\|Ax - b\|_2$ for all $x \in \mathscr{K}_n$, giving an $(n+1) \times n$ least-square problem involving the matrix $\tilde{H}_n$.

Suppose that we **instead** find an approximate solution to $Ax = b$ by finding an $x \in \mathscr{K}_n$ where $b - Ax$ is $\perp \mathscr{K}_n$. Derive a small ($n \times n$ or similar) system of equations that you can solve to find the approximate solution x of this method.

Problem 2: Power method

Suppose A is a diagonalizable matrix with eigenvectors $\mathbf{v}_k$ and eigenvalues λ_k, in decreasing order $|\lambda_1| \geq |\lambda_2| \geq \cdots$. Recall that the power method starts with a random $\mathbf{x}$ and repeatedly computes $\mathbf{x} \leftarrow A\mathbf{x}/\|A\mathbf{x}\|_2$.

(a) Suppose $|\lambda_1| = |\lambda_2| > |\lambda_3|$, but $\lambda_1 \neq \lambda_2$. Explain why the power method will not in general converge.

(b) Give a way obtain λ_1 and λ_2 and $\mathbf{v}_1$ and $\mathbf{v}_2$ from the power method by simply keeping track of the *previous* iteration's vector in addition to the current iteration.

Problem 3: Shifted-inverse iteration

Trefethen, problem 27.5.

Problem 4: Arnoldi

Trefethen, problem 33.2.