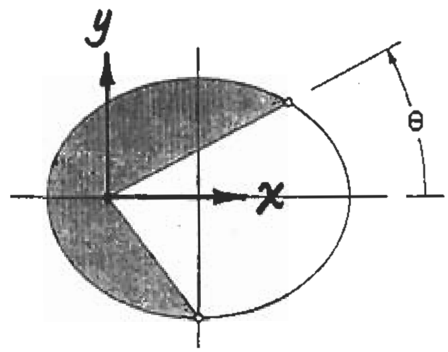
18.330 Problem Set 8 Due in class: Fri 09 Apr 04



(22) Return to that comet from Problem 1, which we saw there would have reached the longitude $\theta = 0.318\ 415\ 710\ 308$ radians half an orbit period later.

Now analyze this problem afresh as one involving the four coupled ODEs

$$\dot{x} = u\ ,\quad \dot{y} = v\ ,\quad \dot{u} = -x/r^3\ ,\quad \dot{v} = -y/r^3$$

let loose at $t = 0$ from $x = 0.6$, $y = -0.8$, $u = \pm 1$, $v = 0$, simple parameters which happen to imply an eccentricity $e = 0.6$ as pictured, and a full orbital period of 2π .

Build yourself a **4-variable RK4** scheme to march that comet exactly a HALF-period π clockwise or counterclockwise along this fine orbit, and report to us the minimum number of (uniform) time steps that this scheme of yours requires in each direction to recover the known final angle θ to some prescribed accuracy like 10^{-6} or 10^{-9} radian.

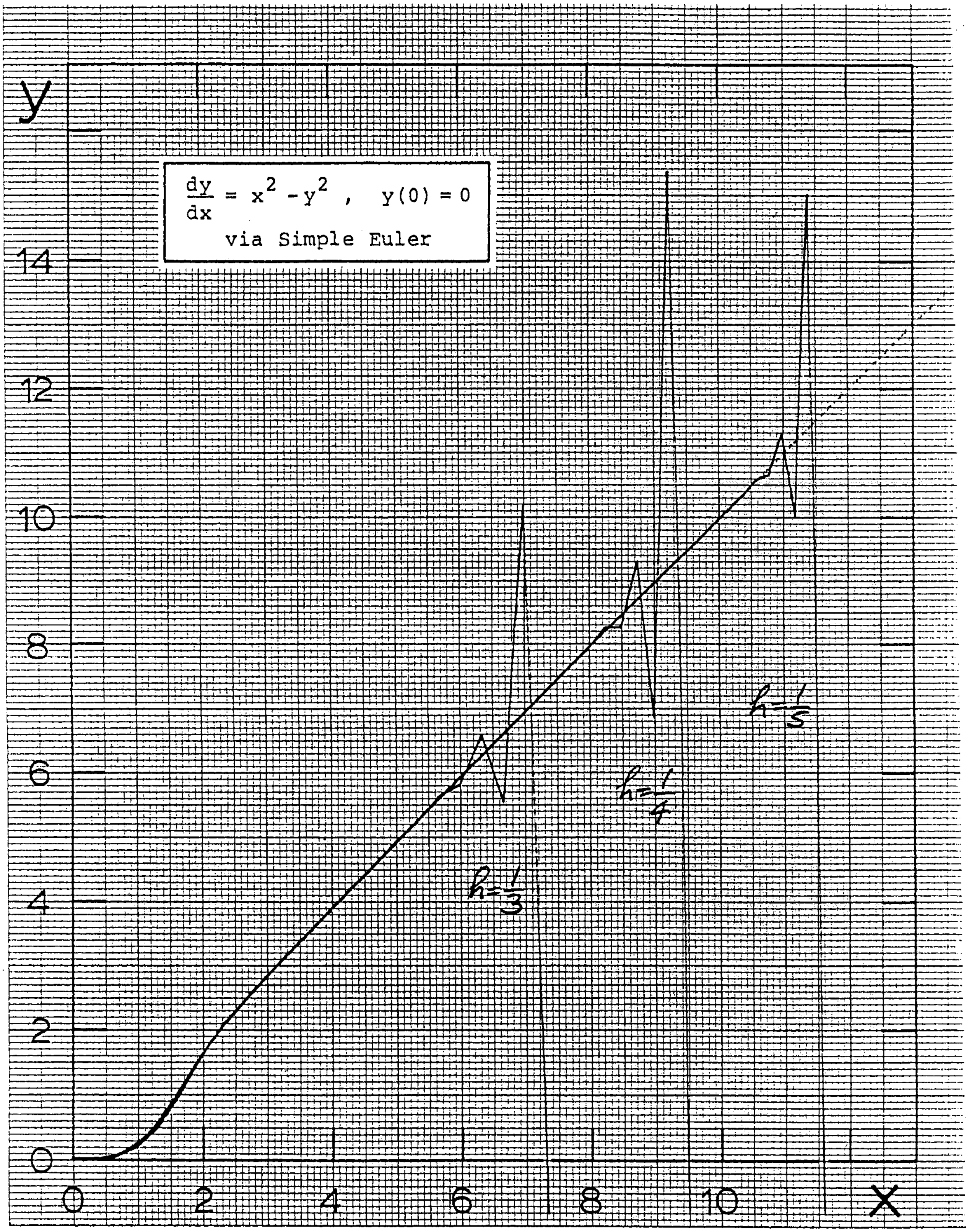
(23) For the test problem $y' = x^2 - y^2$, $y(0) = 0$, explore via some constant step sizes $h = 1/N$ and $N = 2, 4, 8, 16, 32, \ldots$ roughly how far you can proceed toward large positive values of x using first (a) the **simple Euler** and then (b) the **RK4** schemes in two valiant attempts to proceed diagonally upward along the "funnel" before each erupts into a violent hum, much as pictured on the back. And at least for case (a), also confirm theoretically via $y' \approx -ay$ that this disaster occurs just about where you deserved it.

(24) Likewise for $y' = x^2 - y^2$, $y(0) = 0$, build yourself a **Milne predictor and fully-iterated-corrector** scheme like

$$y_{n+1}^{(0)} = y_{n-3} + (4h/3)\ (2f_{n-2} - f_{n-1} + 2f_n)$$

$$y_{n+1}^{(k)} = y_{n-1} + (h/3)\ (f_{n-1} + 4f_n + f_{n+1}^{(k-1)})$$

the latter meant to be repeated for $k = 1, 2, 3, \ldots$ at each time step until no further change in y_{n+1} is perceptible. Initiate this process with values y_0, y_1, y_2, y_3 implied by the terms $y(x) = x^3/3 - x^7/63 + 2x^{11}/2079$ of the known Taylor series, and then use it to demonstrate that the insidious instability of the Milne scheme will here keep you from reaching even rather modest values of x of order 10 or 20 , **no matter how small you choose the stepsize h !**



y
dy/dx = x^2 - y^2 , y(0) = 0
via Simple Euler
14
12
10
8
6
4
2
0
0
2
4
6
8
10
x
h=1/3
h=1/4
h=1/5