Chapter 7

Seismic imaging

Much of the imaging procedure was already described in the previous chapters. An image, or a gradient update, is formed from the imaging condition by means of the incident and adjoint fields. This operation is called migration rather than backprojection in the seismic setting. The two wave equations are solved by a numerical method such as finite differences.

In this chapter, we expand on the structure of the Green's function in variable media. This helps us understand the structure of the forward operator F, as well as migration F^*, in terms of a 2-point traveltime function $\tau(x, y)$. This function embodies the variable-media generalization of the idea that time equals total distance over wave speed, an aspect that was crucial in the expression of backprojection for SAR. Traveltimes are important for "traveltime tomography", or matching of traveltimes for inverting a wave speed profile. Historically, they have also been the basis for Kirchhoff migration, a simplification of (reverse-time) migration.

7.1 Assumptions and vocabulary

In seismic imaging, the forward operator F is called *Born modeling*, and the adjoint/imaging operator F^* is called *migration*. F is sometimes also called *demigration*. We will see in a later section that F^* "undoes" most of F in a kinematic sense. The set of $F_s^* d_s$ indexed by the source s is called *prestack migration*, or prestack depth migration (PSDM) by geophysicists. They call the sum over s in $F^* d = \sum_s F_s^* d_s$ a postmigration stack (not poststack migration!).

A point x inside the Earth is said to be in the *subsurface*. Usually $x = (x', z)$ where x' are the horizontal coordinates, and z is depth. Data space is indexed by $(x_r, , x_s, t)$, where x_r, x_s are usually (but not always) at $z = 0$. Datasets are also called *seismograms*. Time is usually depicted as a vertical axis, pointing *down*.

They also call the source s a *shot*. When the dataset d_s is indexed by shot s, it is called a *shot gather*, or common shot gather (CSG). Alternatively, a dataset can be organized by midpoint $x_m = \frac{x_s + x_r}{2}$ vs. half-offset $h = \frac{x_s - x_r}{2}$ — in that case it is called common midpoint gather (CMP). Since the midpoint does not generally fall on a grid, doing a CMP requires binning[1].

(!) Many physical assumptions are made to keep the algebra simple in this chapter. They are as follows:

1. We use a wave equation that models acoustic waves, rather than elastic waves. All the waves are treated like P waves, hence mistakes will be made with the S waves, as well as mode conversion. Another consequence of using the acoustic simplification is that the medium is assumed to be isotropic, i.e., the waves do not travel at different speeds depending on the direction in which they travel. Laminated rocks are usually anisotropic. Finally, we do not model the (frequency-dependent) dissipation of energy, and dispersion, of the seismic waves.

2. We assume constant-density (single-parameter) acoustics.

3. Boundary conditions are omitted, corresponding to the situation of non-reflecting boundaries. This can be inaccurate in the presence of water-air or rock-air interfaces, better modeled by a Neumann condition (see chapter 1). Free boundaries corresponding to topography (or bathymetry) can be particularly hard to model properly.

4. We assume the source is known, and of the form $f_s(x, t) = \delta(x - x_s)w(t)$. In practice the source needs to be determined or calibrated, usually from a direct arrival. The assumption that the source is a scalar function of x and t in an acoustic wave equation is itself not necessarily accurate (such as in earthquakes, for instance.)

5. Data are assumed to be point samples of the solution of the wave equation, rather than filtered versions thereof (both in space and time).

[1] Which can be a very inaccurate operation from a numerical viewpoint, i.e., as inaccurate as its adjoint, nearest-neighbor interpolation.

Typically, filtered versions of pressure disturbances are measured by microphones.

6. We do not deal with the issue of acquisition noise in data (malfunctioning detectors, ambient seismic noise, incoherent scattering off of structure that are not to be imaged, etc.).

7.2 Kirchhoff modeling and migration

Consider a single, fixed source at location x_s. (If there are several sources, they are handled as in the earlier section on "stacks".) We have already seen how migration F^* is computed with the so-called imaging condition in chapter 4. In that case, F^* is called *reverse-time migration*.

The "Kirchhoff" version of the forward and adjoint operators F and F^* are obtained by using the geometrical optics approximation of the Green's function, from chapter 2. Starting from the Born approximation, we obtain Kirchhoff modeling after a few lines of algebra:

$$(Fm)(x_r,t) = \int a(x_s,x)a(x,x_r)\delta''(t-\tau(x_r,x,x_s))m(x)\,dx,$$

where $\tau(x_r,x,x_s) = \tau(x_r,x) + \tau(x,x_s)$ is the three-point traveltime. The curve/surface $t = \tau(x_r,x,x_s)$ traced in x-space (model space) is called *isochrone*. It is an ellipse/ellipsoid when the background wave speed is uniform.

Passing to the adjoint in a now-familiar manner by equating $\langle d, Fm\rangle = \langle F^*d, m\rangle$, we obtain Kirchhoff migration as

$$(F^*d)(m) = \iint a(x_s,x)a(x,x_r)\delta''(t-\tau(x_r,x,x_s))d(x_r,t)\,dx_r dt.$$

The curve/surface $t = \tau(x_r,x,x_s)$ traced in (x_r,t) space (data space) is called *moveout curve/surface*. It is a hyperbola/hyperboloid when the background wave speed is uniform. Certain references call Kirchhoff migration any backprojection-style formula where either the amplitudes $a(x_s,x)$, $a(x,x_r)$ or the derivatives on the Dirac delta are absent, or both. Both F and F^*, in their "Kirchhoff" version, are generalized Radon transforms.

Historically, Kirchhoff migration (KM) has been very important because it is an explicit formula that requires solving ODEs (for τ, mostly), not PDE. Hence it is computationally much more attractive than reverse-time migration (RTM). KM is still used in optimization schemes where cheap inexact iterations are useful.

7.3 Depth extrapolation

7.4 Extended modeling

7.5 Exercises

1. Show the formula for Kirchhoff migration directly from the imaging condition (4.3) and the geometrical optics approximation of the Green's function.

2. Predict the recorded arrival time, for fixed x_s and as a function of x_r at the surface, of the wave that undergoes a single reflction off a horizontal reflector at depth z. Assume a constant background speed c_0.