Pak

11/19/05

(last class) **Thm** (Graham - Kleitman)

$\alpha : E(K_n) \longleftrightarrow [\binom{n}{2}]$ labelling of edges

$\Rightarrow \exists$ increasing trail of length $\geq n-1$

(this is sharp; lex ordering) $(n \geq 4 ?)$ (nope!)

Def'n $\chi : E(G) \to [k]$ a coloring

χ is swell if every Δ is 1 or 3 colored and χ has at least 2 diff't colors

$\eta(G) =$ min'l # colors for a swell coloring

Thm (Ward - Szabo) (1994)

$\eta(K_n) \geq \sqrt{n} + 1$

Pf: ~~Let ... max # colors in a swell~~

χ a swell coloring w/ r colors, $m =$ max # edges w/ same color ~~adjacent to~~ a vertex

Then $m \cdot r \geq n-1$. Let x be vx w/ max edges all colored c. Then edges w/in $N(x)$ are also colored c, we've got monoch K_{m+1}, $\forall\, y \notin K_{m+1}$, $\forall\, y$-K_{m+1} edges of color $\neq c$ (b/c m max'l + G has ≥ 2 colors)

and all y-K_{m+1} edges are different colors, so $\exists$ at least $(m+1)$ ~~diff't~~ diff't colors $m+2 \leq r \Rightarrow$

~~$(r+2)\, r \geq n-1$~~ $\Rightarrow (r-2)\, r \geq n-1$

$\Rightarrow r^2 - 2r + 1 \geq n \Rightarrow r-1 \geq \sqrt{n}$ ✓

Jukna Ch13 part5

Example: $n = q^2$; q a power of a prime
$r = q+1$. Look at 3-dim vectorspace W
over $\mathbb{F}_q$. Define a graph whose
vertices are lines + edges are planes
containing those lines.

Very very important, v.v. famous,
Thm: (Turán 1940)

p. 135 in MGT 4.2 p 110

Def'n $t_r(n)$ = max # edges in a graph on
n vertices w/out K_{r+1}

$t_r(n) = |T_r(n)|$, where $T_r(n)$ is the Turán
graph $n = r \cdot p + s$ $p \leq s < r$
look at complete r-partite graph whose parts
have size p or $p+1$.

Def'n $ex(H, n)$ = max # edges in H-avoiding graph
on n vertices

Thm (Erdős - Stone, 1946)
$ex(H,n) = \left(1 - \frac{1}{\chi(H)-1} + o(1)\right)\binom{n}{2}$