Pak

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Two more applications of linear algebra

① Packing w/ complete bipartite graphs

$E(G) = \coprod_{i=1} E(H_i)$ "edge decomposition" and $\forall i\ \exists p,q$ s.t. $H_i = K_{p,q}$

Question: What is min'l r?

Thm $r(K_n) \geq n-1$

Pf Let M = adjacency* matrix of $G = K_n$

A_i = " * " " H_i

where the star means lower triangle is all 0's

Then $rk(M) = n-1$ and $rk(A_i) = 1$, and

$rk(A_1 + \dots + A_r) \leq rk(A_1) + \dots + rk(A_r) = r \Rightarrow r \geq n-1$ ✓

② CS Matrix product testing

$A, B \in GL(n,q)$ → $n \times n$ invertible matrices over $\mathbb{F}_q$

$\exists$ Probabilistic algorithm testing if $A \circ B = A \cdot B$ in $O(n^2)$ steps (w/ high prob)

Given A, B, $A \circ B$, let

$v = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}$ $c_i \in \mathbb{F}_q$ v random

Checking $(A \circ B)\, v \overset{?}{=} A(B \cdot v)$ takes $O(n^2)$

Lemma: If $A \circ B \neq A \cdot B$, then single check will fail w/ prob $\geq 1/2$ (actually $1 - 1/q$)

That's because the subspace of v s.t. $A \circ B v = A B v$ is v s.t. $(A \circ B - A \cdot B) v = \bar{0}$, which must have dimension at most $n-1$, so prob of missing it is $1 - q^{n-1}/q^n$

so $|\text{subspace}| \leq q^{n-1}$

Prob gives wrong answer is 1/q, iterate...

Puzzle

K_{10} ~~…~~ want packing w/ Peterson graphs

It's impossible! Pf by linear algebra (no kidding!)

(look at eigenvalues of adjacency matrices...)