HOMEWORK 5 (18.315, FALL 2005)

1) In the *Eventown*, there are $2n$ people and m clubs $A_1, \ldots, A_m$ such that $|A_i|$ and $|A_i \cap A_j|$ are even, for all $1 \le i, j \le m$. Prove that $m \le 2^n$.

2) Compute the probability that a random permutation $\sigma \in S_n$ is an involution: $\sigma^2 = 1$. Compute the probability that two random permutations $\sigma, \omega \in S_n$ commute: $\sigma\omega = \omega\sigma$. Which event is more likely?

3) Try to classify all finite connected planar vertex-transitive graphs. (If you can't find all of them – explain clearly what subclass of them you *can* classify.)

4) Prove or disprove the following result: Every plane triangulation without separating triangles contains a Hamiltonian cycle. Here a triangle is called *separating* if it is a triangle in a graph but not a face.

Important: If you can't figure this out on your own, try to read Whitney's article (see the web page).

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