Time it takes to reach R = 5 cm [salt in water].
Conservation laws and pde:
Higher order (TRANSPORT) effects beyond quasi-equilibrium.
Note difference between heat/diffusion and the other examples. In one case the fluxes are functions of the densities only, in the other derivatives of the densities appear.
   • Important when gradients are not so small.
   • Connection with randomness at microscopic level.
 EXAMPLES:
   • Viscosity in fluids.
   • Slow granular flow in a silo.
   • Preventive driving in traffic flow.

Another example of a higher order transport effect is viscosity: Forces proportional to the flow velocity gradient. REASON: the flow velocity is a macroscopic "average" variable. The same phenomena that causes heat conduction for thermal energy, produces momentum transfer (forces), as well as kinetic energy transfer.

A small peek at the Navier-Stokes equations: to the conservation-law derivation of the Euler equations (#006), add heat conductivity and viscosity. Derive compressible Navier Stokes in 1-D.

TOPIC: Solution of 1st order (scalar) quasilinear equations by characteristics.

Examples: traffic flow and river waves.

Traffic Flow equations. Traffic density, flow and car velocity.

Linearize equation near a constant density and solve $u_t + c_0 * u_x = 0$.
Do it first by separation of variables/Fourier.
Show solution has form $u = f(x - c_0 * t)$.
This works because equation is linear and constant coefficients.
Need approach that will generalize to $u_t + c(u) * u_x = 0$.

But first, note that: for traffic flow u > c and for river flows u < c.
Density waves reach cars from ahead.
River waves move faster than the flow.

Interpret equation as a statement about a directional derivative of the solution in space-time being zero. Leads to the solution being constant along the integral curves for the directional derivatives.

Definition of characteristics and solution by characteristics. Applies to any equation that can be translated into statements about the directional derivatives of the solutions. Such equations are called hyperbolic.

Scalar hyperbolic equations can be reduced to o.d.e.'s along the curves integrating the corresponding directional derivatives.

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