Derive characteristics for the full 1-D isentropic Gas Dynamics.
$\rho_t + u*\rho_x + \rho*u_x = 0$
$u_t + (a^2/\rho)*\rho_x + u*u_x = 0$
where $a^2 = dp/d\rho$.

Write in the form $Y_t + A(Y)*Y_x = 0$, and find the eigenvalues and left eigenvectors for A (namely: solve $L*A = c*L$).

Then $c = u \pm a$, and the characteristic form is:
$\pm(a/\rho)*(d\rho/dt) + du/dt = 0$
            along $dx/dt = u \pm a$.

Two sets of characteristics, which interact and couple. Situation similar to $u_{tt} - u_{xx} + V(u)$, but more complicated: Now the characteristic speed is no longer constant. Characteristics in the same family may cross, leading to shocks.

Introduce $h = h(\rho)$ by property $dh/d\rho = a/\rho$.
Show for ideal gas $h = 2*a/(\gamma-1)$.

Then $d/dt\ (u \pm h) = 0$ along $dx/dt = u \pm a$.
i.e $(u \pm h)$ is constant along characteristics.

Show how this, in principle, determines the solution. At each point in space time two characteristics [C+ and C-], each carrying information from a different part of the initial data, which combined gives the solution at the point. But now the characteristics are neither straight, nor can we solve for them explicitly, because they interact with each other.

Assume u-h = constant for initial data. Then characteristics yield u-h = L = constant, as long as characteristic form applies.

Hence $u = h + L = U(\rho)$ is a function of ρ only. The equations then reduce to
$d/dt\ (u+h) = 0$ along $dx/dt = u+a$.

That is
        $(U+h)_t + (u+a)*(U+h)_x = 0$,

which is a first order equation of the same type as Traffic Flow and River Flows. Hence characteristics can cross and once this happens we need to re-examine the physics to see what to do beyond breakdown.

In this case, again, shocks are the appropriate answer.

Shocks in Gas Dynamics:
 • Rankine-Hugoniot jump conditions (and graphical interpretation).

- Entropy conditions.

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