Homework 1; due Tuesday, Sept. 17

1. Write a complete proof of Theorem 1.1. (i.e. fill the gaps left in the lecture notes)

2* (slightly harder). Prove Theorem 1.2.

3. Calculate $\int_0^\pi \sin^n(x)dx$ for nonnegative integers n, using integration by parts. Then apply stationary phase to this integral, and discover a formula for π (the so called Wallis formula).

4. Prove that if the potential for a moving particle is $U(q) = -B(q,q)$, where B is a nonnegative definite symmetric bilinear form on a Euclidean space V, then for any $q_1, q_2 \in V$ and $t_1 < t_2 \in \mathbb{R}$ there exists a unique solution of the Newton equation with $q(t_1) = q_1$ and $q(t_2) = q_2$. Show that it provides not only an extremum but also a minimum for the action with these boundary conditions. What happens if B is not nonnegative?