Problem set 4

This was late going up, so anyone who really needs the time can wait until Thursday October 14 to hand in solutions. If you give them to me on Tuesday October 12 I should be able to mark them by October 14.

Problems 29, 40, 41 from the notes and the following additional problem:

[Note that there was a nasty typo in this question when I first put it up.]

Problem 1 If $u \in \mathcal{S}'(\mathbb{R})$ show that there exists $v \in \mathcal{S}'(\mathbb{R})$ satisfying

$$\frac{d}{dx}v = u. \tag{5}$$

Hint:- Show that if $\phi \in \mathcal{C}_c^\infty(\mathbb{R})$ has $\int_{\mathbb{R}} \phi(t)dt = 1$ then any $\psi \in \mathcal{S}(\mathbb{R})$ can be written in the form

$$\psi = c\phi + \frac{d}{dx}(A\psi), \tag{6}$$

Where

$$A : \mathcal{S}(\mathbb{R}) \longrightarrow \mathcal{S}(\mathbb{R}) \tag{7}$$

is a continuous linear operator.