Problem Set 10

1. Let $K : [0,1] \times [0,1] \to \mathbb{R}$ be a continuous function. Show that if $f : [0,1] \to \mathbb{R}$ is Riemann-integrable, the function

$$g(x) = \int_0^1 K(x,y)f(y)\,dy$$

makes sense and is in fact continuous. *(Hint: you may find the compactness of* $[0,1] \times [0,1]$ *useful; and also the Riemann-integrability of* $|f|$*.)*(5 points)

2. Prove that if $f, g : [a,b] \to \mathbb{R}$ are Riemann-Stieltjes integrable (for some α), then so is the function $\max(f,g)$. **The solution for this problem should be written up carefully in LaTeX.** (3 points)

3. (i) Prove that if $f : [a,b] \to \mathbb{R}$ is a continuous function and not everywhere zero, then $\int_a^b f(x)^2\,dx > 0$.
(ii) Using that, prove that if $f : [a,b] \to \mathbb{R}$ is a continuous function and $\int_a^b x^n f(x)\,dx = 0$ for all $n \geq 0$, then f is everywhere zero. (5 points)

Total: 5+3+5 = 13 points.

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