Practice Quiz 1

18.100B R2 Fall 2010

Closed book, no calculators.

YOUR NAME: _____

This is a 30 minute in-class exam. No notes, books, or calculators are permitted. Point values are indicated for each problem. Do all the work on these pages.

GRADING	
1. _____	/15
2. _____	/20
3. _____	/15
4. _____	/20
TOTAL	/70

Problem 1. [5+5+5 points]

(a) Write down the definition of compactness in an arbitrary metric space.

(b) Prove that finite sets are always compact.

(c) Give an example of an infinite set that is not compact. (Show why it does not satisfy your definition in (a))

Problem 2. [10+10 points]

(a) Let A and B be countable sets. Prove that $A \cup B$ is countable and that $A \cap B$ is at most countable, using the definition of countability.

(b) Consider two subsets $S, T \subset \mathbb{R}$ and their sum

$$S + T := \{s + t \,|\, s \in S, t \in T\} \subset \mathbb{R}.$$

Show (from the definition of a supremum) that $\sup(S + T) = \sup S + \sup T$.

Problem 3. [5+5+5 points] Consider $X = \{0\} \cup \{\frac{1}{n} \mid n \in \mathbb{N}\}$ as metric space with metric induced from the standard metric of $\mathbb{R}$.

a) What are the limit points of X?

b) What are the closed subsets of X?

c) What are the compact subsets of X? Why?

Problem 4. [20 points: $+4$ for each correct, -4 for each incorrect; no proofs required.]

a) For any open set $A \subset \mathbb{R}$, we have $int(\bar{A}) = A$.

TRUE FALSE

b) Let V be the set of all functions $f : [0, 1] \to \mathbb{R}$, and define $d(f, g) = |f(0) - g(0)|$. Then (V, d) is a metric space.

TRUE FALSE

c) If X is a compact metric space and $E \subset X$ is not compact, then E is not closed.

TRUE FALSE

d) The set $\{(x, y) \in \mathbb{R}^2 | x + y \in \mathbb{Q}\}$ is countable.

TRUE FALSE

e) The set $\{(x, y) \in \mathbb{R}^2 | x + y \in \mathbb{Q}, x - y \in \mathbb{Q}\}$ is countable.

TRUE FALSE

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18.100B Analysis I
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