

Contents

Preface xi

- 1. Real Numbers and Monotone Sequences 1**
 - 1.1 Introduction; Real numbers 1
 - 1.2 Increasing sequences 3
 - 1.3 Limit of an increasing sequence 4
 - 1.4 Example: the number e 5
 - 1.5 Example: the harmonic sum and Euler's number γ 8
 - 1.6 Decreasing sequences; Completeness property 10
- 2. Estimations and Approximations 17**
 - 2.1 Introduction; Inequalities 17
 - 2.2 Estimations 18
 - 2.3 Proving boundedness 20
 - 2.4 Absolute values; estimating size 21
 - 2.5 Approximations 24
 - 2.6 The terminology "for n large" 27
- 3. The Limit of a Sequence 35**
 - 3.1 Definition of limit 35
 - 3.2 Uniqueness of limits; the K - ϵ principle 38
 - 3.3 Infinite limits 40
 - 3.4 Limit of a^n 42
 - 3.5 Writing limit proofs 43
 - 3.6 Some limits involving integrals 44
 - 3.7 Another limit involving an integral 45
- 4. The Error Term 51**
 - 4.1 The error term 51
 - 4.2 The error in the geometric series; Applications 52
 - 4.3 A sequence converging to $\sqrt{2}$: Newton's method 53
 - 4.4 The sequence of Fibonacci fractions 56
- 5. Limit Theorems for Sequences 61**
 - 5.1 Limits of sums, products, and quotients 61
 - 5.2 Comparison theorems 64
 - 5.3 Location theorems 67
 - 5.4 Subsequences; Non-existence of limits 68
 - 5.5 Two common mistakes 71

- 6. The Completeness Property 78**
 - 6.1 Introduction; Nested intervals 78
 - 6.2 Cluster points of sequences 80
 - 6.3 The Bolzano-Weierstrass theorem 82
 - 6.4 Cauchy sequences 83
 - 6.5 Completeness property for sets 86

- 7. Infinite Series 94**
 - 7.1 Series and sequences 94
 - 7.2 Elementary convergence tests 97
 - 7.3 The convergence of series with negative terms 100
 - 7.4 Convergence tests: ratio and n -th root tests 102
 - 7.5 The integral and asymptotic comparison tests 104
 - 7.6 Series with alternating signs: Cauchy's test 106
 - 7.7 Rearranging the terms of a series 107

- 8. Power Series 114**
 - 8.1 Introduction; Radius of convergence 114
 - 8.2 Convergence at the endpoints; Abel summation 117
 - 8.3 Operations on power series: addition 119
 - 8.4 Multiplication of power series 120

- 9. Functions of One Variable 125**
 - 9.1 Functions 125
 - 9.2 Algebraic operations on functions 127
 - 9.3 Some properties of functions 128
 - 9.4 Inverse functions 131
 - 9.5 The elementary functions 133

- 10. Local and Global Behavior 137**
 - 10.1 Intervals; estimating functions 137
 - 10.2 Approximating functions 141
 - 10.3 Local behavior 143
 - 10.4 Local and global properties of functions 145

- 11. Continuity and Limits of Functions 151**
 - 11.1 Continuous functions 151
 - 11.2 Limits of functions 155
 - 11.3 Limit theorems for functions 158
 - 11.4 Limits and continuous functions 162
 - 11.5 Continuity and sequences 155

- 12. The Intermediate Value Theorem 172**
 - 12.1 The existence of zeros 172
 - 12.2 Applications of Bolzano's theorem 175
 - 12.3 Graphical continuity 178
 - 12.4 Inverse functions 179

Contents

13. Continuous Functions on Compact Intervals	185
13.1 Compact intervals	185
13.2 Bounded continuous functions	186
13.3 Extremal points of continuous functions	187
13.4 The mapping viewpoint	189
13.5 Uniform continuity	190
14. Differentiation: Local Properties	196
14.1 The derivative	196
14.2 Differentiation formulas	200
14.3 Derivatives and local properties	202
15. Differentiation: Global Properties	210
15.1 The mean-value theorem	210
15.2 Applications of the mean-value theorem	212
15.3 Extension of the mean-value theorem	214
15.4 L'Hospital's rule for indeterminate forms	215
16. Linearization and Convexity	222
16.1 Linearization	222
16.2 Applications to convexity	225
17. Taylor Approximation	231
17.1 Taylor polynomials	231
17.2 Taylor's theorem with Lagrange remainder	233
17.3 Estimating error in Taylor approximation	235
17.4 Taylor series	236
18. Integrability	241
18.1 Introduction; Partitions	241
18.2 Integrability	242
18.3 Integrability of monotone and continuous functions	244
18.4 Basic properties of integrable functions	246
19. The Riemann Integral	251
19.1 Refinement of partitions	251
19.2 Definition of the Riemann integral	253
19.3 Riemann sums	255
19.4 Basic properties of integrals	257
19.5 The interval addition property	258
19.6 Piecewise continuous and monotone functions	260
20. Derivatives and Integrals	269
20.1 The first fundamental theorem of calculus	269
20.2 Existence and uniqueness of antiderivatives	270
20.3 Other relations between derivatives and integrals	274
20.4 The logarithm and exponential functions	276
20.5 Stirling's formula	278
20.6 Growth rate of functions	280

- 21. Improper Integrals 290**
 - 21.1 Basic definitions 290
 - 21.2 Comparison theorems 292
 - 21.3 The gamma function 295
 - 21.4 Absolute and conditional convergence 298
- 22. Sequences and Series of Functions 305**
 - 22.1 Pointwise and uniform convergence 305
 - 22.2 Criteria for uniform convergence 310
 - 22.3 Continuity and uniform convergence 312
 - 22.4 Integration term-by-term 314
 - 22.5 Differentiation term-by-term 316
 - 22.6 Power series and analytic functions 318
- 23. Infinite Sets and the Lebesgue Integral 329**
 - 23.1 Introduction; infinite sets 329
 - 23.2 Sets of measure zero 333
 - 23.3 Measure zero and Riemann-integrability 335
 - 23.4 Lebesgue integration 338
- 24. Continuous Functions on the Plane 347**
 - 24.1 Introduction; Norms and distances in \mathbb{R}^2 347
 - 24.2 Convergence of sequences 349
 - 24.3 Functions on \mathbb{R}^2 351
 - 24.4 Continuous functions 352
 - 24.5 Limits and continuity 354
 - 24.6 Compact sets in \mathbb{R}^2 355
 - 24.7 Continuous functions on compact sets in \mathbb{R}^2 356
- 25. Point-sets in the Plane 364**
 - 25.1 Closed sets in \mathbb{R}^2 364
 - 25.2 Compactness theorem in \mathbb{R}^2 367
 - 25.3 Open sets 368
- 26. Integrals with a Parameter 375**
 - 26.1 Integrals depending on a parameter 375
 - 26.2 Differentiating under the integral sign 377
 - 26.3 Changing the order of integration 380
- 27. Differentiating Improper Integrals 386**
 - 27.1 Introduction 386
 - 27.2 Pointwise vs. uniform convergence of integrals 387
 - 27.3 Continuity theorem for improper integrals 390
 - 27.4 Integrating and differentiating improper integrals 391
 - 27.5 Differentiating the Laplace transform 393

Appendix

- A. Sets, Numbers, and Logic 399**
 - A.0 Sets and numbers 399
 - A.1 If-then statements 403
 - A.2 Contraposition and indirect proof 406
 - A.3 Counterexamples 408
 - A.4 Mathematical induction 411
 - B. Quantifiers and Negation 418**
 - B.1 Introduction; Quantifiers 418
 - B.2 Negation 421
 - B.3 Examples involving functions 423
 - C. Picard's Method 427**
 - C.1 Introduction 427
 - C.2 The Picard iteration theorems 428
 - C.3 Fixed points 430
 - D. Applications to Differential Equations 434**
 - D.1 Introduction 434
 - D.2 Discreteness of the zeros 435
 - D.3 Alternation of zeros 437
 - D.4 Reduction to normal form 439
 - D.5 Comparison theorems for zeros 440
 - E. Existence and Uniqueness of ODE Solutions 445**
 - E.1 Picard's method of successive approximations 445
 - E.2 Local existence of solutions to $y' = f(x, y)$ 447
 - E.3 The uniqueness of solutions 450
 - E.4 Extending the existence and uniqueness theorems 452
- Index 455**

MIT OpenCourseWare
<http://ocw.mit.edu>

18.100A Introduction to Analysis
Fall 2012

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.