18.06SC Unit 2 Exam

1 (24 pts.) Suppose q_1, q_2, q_3 are orthonormal vectors in $\mathbb{R}^3$. Find **all possible values** for these 3 by 3 determinants and explain your thinking in 1 sentence each.

(a) $\det\begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix} =$

(b) $\det\begin{bmatrix} q_1+q_2 & q_2+q_3 & q_3+q_1 \end{bmatrix} =$

(c) $\det\begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$ times $\det\begin{bmatrix} q_2 & q_3 & q_1 \end{bmatrix} =$

2 (24 pts.) Suppose we take measurements at the 21 equally spaced times $t = -10, -9, \ldots, 9, 10$. All measurements are $b_i = 0$ except that $b_{11} = 1$ at the middle time $t = 0$.

(a) Using least squares, what are the best $\widehat{C}$ and $\widehat{D}$ to fit those 21 points by a straight line $C + Dt$?

(b) You are projecting the vector b onto what subspace? (*Give a basis.*) Find a nonzero vector perpendicular to that subspace.

3 (9 + 12 + 9 pts.) The Gram-Schmidt method produces orthonormal vectors q_1, q_2, q_3 from independent vectors a_1, a_2, a_3 in $\mathbb{R}^5$. Put those vectors into the columns of 5 by 3 matrices Q and A.

(a) Give formulas using Q and A for the projection matrices P_Q and P_A onto the column spaces of Q and A.

(b) ***Is $P_Q = P_A$ and why ? What is P_Q times Q ? What is*** $\det P_Q$ ***?***

(c) Suppose a_4 is a new vector and a_1, a_2, a_3, a_4 are independent. Which of these (if any) is the new Gram-Schmidt vector q_4 ? (P_A and P_Q from above)

1. $\dfrac{P_Q a_4}{\|P_Q a_4\|}$ **2.** $\dfrac{a_4 - \frac{a_4^{\mathrm{T}} a_1}{a_1^{\mathrm{T}} a_1} a_1 - \frac{a_4^{\mathrm{T}} a_2}{a_2^{\mathrm{T}} a_2} a_2 - \frac{a_4^{\mathrm{T}} a_3}{a_3^{\mathrm{T}} a_3} a_3}{\| \text{ norm of that vector } \|}$ **3.** $\dfrac{a_4 - P_A a_4}{\|a_4 - P_A a_4\|}$

4 (22 pts.) Suppose a 4 by 4 matrix has the same entry $\times$ throughout its first row and column. The other 9 numbers could be anything like $1, 5, 7, 2, 3, 99, \pi, e, 4$.

$$A = \begin{bmatrix} \times & \times & \times & \times \\ \times & \text{any numbers} \\ \times & \text{any numbers} \\ \times & \text{any numbers} \end{bmatrix}$$

(a) The determinant of A is a polynomial in $\times$. What is the largest possible degree of that polynomial? **Explain your answer**.

(b) If those 9 numbers give the identity matrix I, what is $\det A$? Which values of $\times$ give $\det A = 0$?

$$A = \begin{bmatrix} \times & \times & \times & \times \\ \times & 1 & 0 & 0 \\ \times & 0 & 1 & 0 \\ \times & 0 & 0 & 1 \end{bmatrix}$$

18.06SC Linear Algebra
Fall 2011