Studio 3

18.05 Spring 2014

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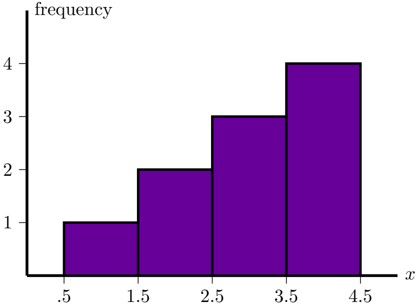


Concept questions

Suppose X is a continuous random variable.

a) What is $P(a \leq X \leq a)$?

b) What is $P(X = 0)$?

c) Does $P(X = 2) = 0$ mean X never equals 2?

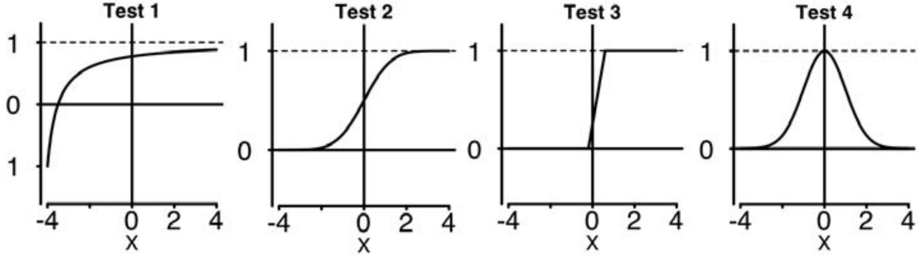
answer: a) 0

b) 0

c) No. For a continuous distribution any single value has probability 0. Only a range of values has non-zero probability.

Concept question

Which of the following are graphs of valid cumulative distribution functions?



Add the numbers of the valid cdf's and click that number.
answer: Test 2 and Test 3.

Solution

Test 1 is not a cdf: it takes negative values, but probabilities are positive.

Test 2 is a cdf: it increases from 0 to 1.

Test 3 is a cdf: it increases from 0 to 1.

Test 4 is not a cdf: it decreases. A cdf must be non-decreasing since it represents *accumulated* probability.

Exponential Random Variables

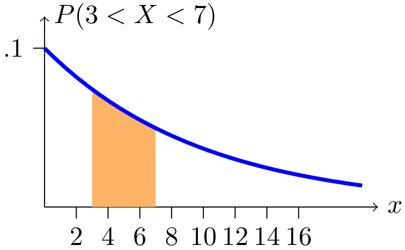
Parameter: λ (called the rate parameter).

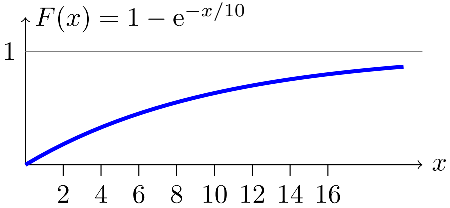
Range: $[0, \infty)$.

Notation: exponential(λ) or exp(λ).

Density: $f(x) = \lambda e^{-\lambda x}$ for $0 \leq x$.

Models: Waiting time





Continuous analogue of geometric distribution –memoryless!

Uniform and Normal Random Variables

Uniform: $U(a, b)$ or uniform(a, b)
Range: $[a, b]$
PDF: $f(x) = \dfrac{1}{b - a}$

Normal: $\mathrm{N}(\mu, \sigma^2)$
Range: $(-\infty, \infty]$
PDF: $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-(x-\mu)^2/2\sigma^2}$

http://mathlets.org/mathlets/probability-distributions/

Table questions

Open the applet

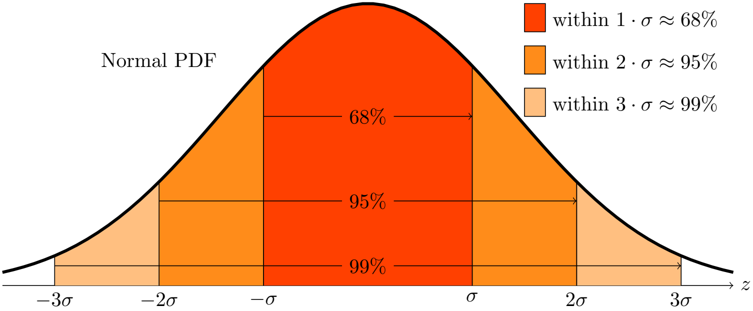
http://mathlets.org/mathlets/probability-distributions/

1. For the **standard normal** distribution $N(0, 1)$ how much probability is within 1 of the mean? Within 2? Within 3?

2. For $N(0, 3^2)$ how much probability is within σ of the mean? Within 2σ? Within 3σ.

3. Does changing μ change your answer to problem 2?

Normal probabilities



Rules of thumb:

$P(-1 \leq Z \leq 1) \approx .68,$
$P(-2 \leq Z \leq 2) \approx .95,$
$P(-3 \leq Z \leq 3) \approx .997$

Download R script

Download studio3.zip and unzip it into your 18.05 working directory.

Open studio3.r in RStudio.

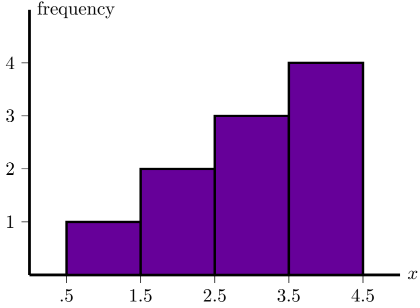
Histograms

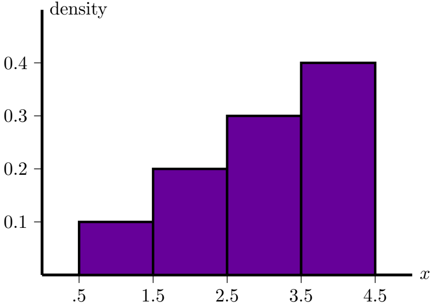
Will discuss in more detail in class 6.

Made by ‘binning’ data.

Frequency: height of bar over bin = # of data points in bin.

Density: area of bar over bin is proportional to # of data points in bin. Total area of a density histogram is 1.





Histograms of averages of exp(1)

1. Generate a frequency histogram of 1000 samples from an exp(1) random variable.

2. Generate a density histogram for the average of 2 independent exp(1) random variable.

3. Using `rexp()`, `matrix()` and `colMeans()` generate a density histogram for the average of 50 independent exp(1) random variables. Make 10000 sample averages and use a binwidth of .1 for this. Look at the spread of the histogram.

4. Superimpose a graph of the pdf of $N(1, 1/50)$ on your plot in problem 3. (Remember the second parameter in N is σ^2.)

Code for the solutions is at

https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/studio-resources/studio3.zip