Introduction

Up to now we have handled systems analytically, concentrating on a procedure for solving linear systems with constant coefficients. In this session, we consider methods for sketching graphs of the solutions.

The emphasis is on the word *sketching*. Computers do the work of drawing reasonably accurate graphs. Here we want to see how to get quick qualitative information about the graph, without having to actually calculate points on it. These graphs of the solutions (also called the trajectories of the system) are called of *phase portraits.*

In this session we consider 2×2 linear homogeneous systems $\mathbf{x}' = A\mathbf{x}$. In a later session we extend this program, known as the phase-plane analysis, to more general non-linear 2×2 DE systems.

The analysis of the linear case will be the foundation for the more general program, so it is very important that we understand this case well. For that reason, this session is somewhat long, we wish to have the linear case well worked out in detail so that we can refer back to it later as needed.

The Eigenvalues Rule

There are a lot of details in this session. The one key fact tying them all together is the eignenvalues rule:

We classify the linear phase portraits according to the eigenvalues of the matrix A.