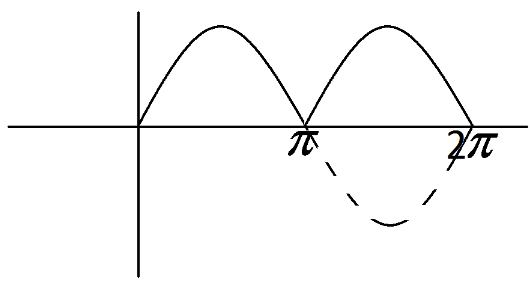
Part I Problems and Solutions

Problem 1: *Find the smallest period for each of the following:*
a) $\sin \pi t/3$
b) $|\sin t|$
c) $\cos^2 3t$

Solution: For the functions $\sin \omega t$, $\cos \omega t$, the *frequency* is ω, and the the frequency and the period are related by (frequency) $\times$ (period) $= 2\pi$. Applying this gives:

a) for $\sin \pi t/3$, $\frac{\pi}{3} \cdot P = 2\pi \to P = 6$.

b) for $|\sin t|$, the period $P = \pi$.
(Check: $|\sin(t+\pi)| = |-\sin(t)| = |\sin t|$.)



c) for $\cos^2 3t$, note that $\cos 3t$ has period $\frac{2\pi}{3}$. Thus (analogously to $|\sin t|$), $(\cos 3t)^2$ has period $\frac{1}{2}\frac{2\pi}{3} = \frac{\pi}{3}$.
(Check: $(\cos 3(t+\frac{\pi}{3}))^2 = (\cos(3t+\pi))^2 = (-\cos(3t))^2 = (\cos(3t))^2$.

Problem 2:

Find the Fourier series of the function $f(t)$ of period 2π which is given over the interval $-\pi < t \leq \pi$ by

$$f(t) = \begin{cases} 0, & -\pi < t \leq 0 \\ 1, & 0 < t \leq \pi \end{cases}$$

Solution:

$$\begin{aligned}
a_n &= \frac{1}{\pi}\int_0^{\pi} \cos nt\, dt = \left.\frac{\sin nt}{\pi n}\right]_0^{\pi} = 0 \\
a_0 &= \frac{1}{\pi}\int_0^{\pi} dt = 1 \\
b_n &= \frac{1}{\pi}\int_0^{\pi} \sin nt\, dt = \left.\frac{-\cos nt}{\pi n}\right]_0^{\pi} = \frac{-(-1)^n - (-1)}{n\pi} \\
&= \frac{1-(-1)^n}{n\pi} = \begin{cases} 0 & n \text{ even} \\ \frac{2}{n\pi} & n \text{ odd} \end{cases}
\end{aligned}$$

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n \text{ odd}} \frac{1}{n}\sin nt = \frac{1}{2} + \frac{2}{\pi}\left(\sin t + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \cdots\right)$$