Part I Problems and Solutions

Problem 1: *a) Find the periodic solution to*

$$x''' + x = 2\cos t$$

in amplitude-phase form.

b) What is the gain and the phase lag ?

Solution: a) characteristic polynomial $p(s) = s^3 + 1$;

complex replacement: $z''' + z = 2e^{it}, \quad x = \text{Re}(z)$

ERF: $p(i) = i^3 + 1 = -i + 1$;

$$z_p = \frac{2}{p(i)} e^{it}$$

$\frac{1}{p(i)} = \frac{1}{1-i} = \frac{1}{\sqrt{2}} e^{i\frac{\pi}{4}}$

$z_p = \frac{2}{\sqrt{2}} e^{i\frac{\pi}{4}} \Rightarrow x_p = \sqrt{2}\cos(t + \frac{\pi}{4})$

b) Gain $= | \frac{1}{p(i)} | = \frac{1}{\sqrt{2}}$;

Phase lag $\phi = -\frac{\pi}{4}$

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