18.034 Honors Differential Equations
Spring 2009

18.034 Recitation: May 14, 2009

1. Study the phase portraits of the systems

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 1 & \epsilon \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

and

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 0 & -1 \\ \epsilon & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

2. Consider

$$x' = y - x(x^2 + y^2), \qquad y' = -x - y(x^2 + y^2).$$

(a) Find the critical point.
(b) Determined the stability of the linear approximation at $(0, 0)$.
(c) Determined the stability of $(0, 0)$.
(d) Repeat for

$$x' = y + x(x^2 + y^2), \qquad y' = -x - y(x^2 + y^2).$$

3. In the competitive system

$$x' = x(k - ax - by), \qquad y' = y(m - cx - dy), \qquad k, m, a, b, c, d > 0$$

if the lines $ax + by = k$ and $cx + dy = m$ do not intersect in the first quadrangle $x, y > 0$ find the limit set.

4. If $(x(t), y(t))$ is a solution of the predator-prey equations

$$x' = x(-k + by), \qquad y' = y(m - cx), \qquad k, m, b, c > 0$$

of period $T > 0$, show that

$$\frac{1}{T}\int_0^T x(t)dt = \frac{m}{c}, \qquad \frac{1}{T}\int_0^T y(t)dt = \frac{k}{b}.$$

5. (a) Show that the differential equation

$$x'' + (x^2 + 2(x')^2 - 1)x' + x = 0$$

has a nontrivial periodic solution.
(b) Show that the system of differential equations

$$x' = x + y^2 + x^3, \qquad y' = -x + y + yx^2$$

has no nontrivial periodic solution.