18.03 Recitation 25, May 11, 2010

Autonomous systems

Write x for the population of bugs (in some convenient units), and y for the population of birds. Birds eat bugs, and the two together satisfy the nonlinear autonomous system

$$\begin{cases} \dot{x} &= (2-x-y)x \\ \dot{y} &= (x-1)y \end{cases}$$

(so that in the absence of birds, the bug population grows logistically, and in the absence of bugs, the birds die out exponentially).

1. Find all the critical points of this system.

2. Find the linearization at each critical point, and sketch the trajectories near them.

3. Locate the phase lines of the bugs-only equation and of the birds-only equation on the (bugs,birds) plane. They form part of the phase portrait of the (bugs,birds) system. Use this and the work from **(2)** to sketch the phase portrait of the (bugs,birds) system.

4. Pick some positive initial values of $x(t)$ and $y(t)$ and use your phase portrait to sketch the graphs of $x(t)$ and $y(t)$. As t gets large, x and y converge to limiting values, exponentially. At what exponential rate? Do they overshoot/undershoot the limiting value, and oscillate around it? If so, what it the pseudoperiod of the oscillation?

5. Now malathion is introduced in an attempt to reduce the bug population. This reduces the rate of reproduction of both species, so the new system is given by

$$\begin{cases} \dot{x} &= (2-x-y-a)x \\ \dot{y} &= (x-1-b)y \end{cases}$$

for certain small positive constants a, b. What happens to the critical point for which both x and y are positive? Is this measure successful in reducing the bug population?

18.03 Differential Equations
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