18.03 Recitation 19, April 15, 2010

Laplace transform II

Rules for the Laplace transform

Definition:	$\mathcal{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}\,dt$ for $\mathrm{Re}\,(s) \gg 0$
Linearity:	$\mathcal{L}[af(t) + bg(t)] = aF(s) + bG(s)$
$\mathcal{L}^{-1}$:	$F(s)$ essentially determines $f(t)$
s-shift rule:	$\mathcal{L}[e^{rt}f(t)] = F(s-r)$
s-derivative rule:	$\mathcal{L}[tf(t)] = -F'(s)$
t-derivative rule:	$\mathcal{L}[f'(t)] = sF(s)$; $\mathcal{L}[f^{(n)}(t)] = s^nF(s)$
t-derivative rule:	If $f(t)$ has no jumps for $t > 0$ then $\mathcal{L}[f_r'(t)] = sF(s) - f(0+)$

$$\mathcal{L}[1] = \frac{1}{s}, \quad \mathcal{L}[\delta(t)] = 1$$
$$\mathcal{L}[e^{rt}] = \frac{1}{s-r}, \quad \mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$
$$\mathcal{L}[\cos(\omega t)] = \frac{s}{s^2+\omega^2}, \quad \mathcal{L}[\sin(\omega t)] = \frac{\omega}{s^2+\omega^2}$$
$$\mathcal{L}[t\sin(\omega t)] = \frac{2\omega s}{(s^2+\omega^2)^2}, \quad \mathcal{L}[t\cos(\omega t)] = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

1. Find the unit impulse response for $D + 3I$.

2. Find the solution to $\dot{x} + 3x = e^{-t}$ with rest initial condition (so $x(0) = 0$) using Laplace transform.

3. Find the unit impulse response for $D^3 + D$ using Laplace transform.

4. (a) Find the solution to $\dot{x}_r + 3x = 1$ such that $x(0+) = 2$ using the formula for $\mathcal{L}[f_r'(t)]$.

(b) The jump in value of $x(t)$ at $t = 0$ can be created by adding $2\delta(t)$ to the right hand side of the equation. Using the expression for $\mathcal{L}[f'(t)]$, find the solution to $\dot{x} + 3x = 1 + 2\delta(t)$ such that $x(0-) = 0$.