

18.03 Recitation 19, April 15, 2010

Laplace transform II

Rules for the Laplace transform

Definition: $\mathcal{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st} dt$ for $\operatorname{Re}(s) \gg 0$

Linearity: $\mathcal{L}[af(t) + bg(t)] = aF(s) + bG(s)$

\mathcal{L}^{-1} : $F(s)$ essentially determines $f(t)$

s -shift rule: $\mathcal{L}[e^{rt}f(t)] = F(s - r)$

s -derivative rule: $\mathcal{L}[tf(t)] = -F'(s)$

t -derivative rule: $\mathcal{L}[f'(t)] = sF(s)$; $\mathcal{L}[f^{(n)}(t)] = s^n F(s)$

t -derivative rule: If $f(t)$ has no jumps for $t > 0$ then

$$\mathcal{L}[f'_r(t)] = sF(s) - f(0+)$$

$$\begin{aligned}\mathcal{L}[1] &= \frac{1}{s}, & \mathcal{L}[\delta(t)] &= 1 \\ \mathcal{L}[e^{rt}] &= \frac{1}{s-r}, & \mathcal{L}[t^n] &= \frac{n!}{s^{n+1}} \\ \mathcal{L}[\cos(\omega t)] &= \frac{s}{s^2 + \omega^2}, & \mathcal{L}[\sin(\omega t)] &= \frac{\omega}{s^2 + \omega^2} \\ \mathcal{L}[t \sin(\omega t)] &= \frac{2\omega s}{(s^2 + \omega^2)^2}, & \mathcal{L}[t \cos(\omega t)] &= \frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}\end{aligned}$$

1. Find the unit impulse response for $D + 3I$.
2. Find the solution to $\dot{x} + 3x = e^{-t}$ with rest initial condition (so $x(0) = 0$) using Laplace transform.
3. Find the unit impulse response for $D^3 + D$ using Laplace transform.
4. (a) Find the solution to $\dot{x}_r + 3x = 1$ such that $x(0+) = 2$ using the formula for $\mathcal{L}[f'_r(t)]$.
(b) The jump in value of $x(t)$ at $t = 0$ can be created by adding $2\delta(t)$ to the right hand side of the equation. Using the expression for $\mathcal{L}[f'(t)]$, find the solution to $\dot{x} + 3x = 1 + 2\delta(t)$ such that $x(0-) = 0$.

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