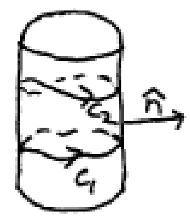
Extended Stokes' Theorem

Let $\mathbf{F} = \langle 2xz + 2y, 2yz + 2yx, x^2 + y^2 + z^2 \rangle$. Take C_1 and C_2 two curves going around the circular cylinder of radius a as shown. Show $\oint_{C_1} \mathbf{F} \cdot d\mathbf{r} = \oint_{C_2} \mathbf{F} \cdot d\mathbf{r}$.



<u>Answer:</u> We easily compute $\text{curl}\mathbf{F} = (2y - 2x)\mathbf{k} \Rightarrow \text{curl}\mathbf{F} \cdot \mathbf{n} = 0$, where $\mathbf{n}$ is the normal to the cylinder. Let S be the part of the cylinder between C_1 and C_2 then Stokes' theorem says

$$\oint_{C_1 - C_2} \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl}\mathbf{F} \cdot \mathbf{n}\, dS = 0. \Rightarrow \oint_{C_1} \mathbf{F} \cdot d\mathbf{r} = \oint_{C_2} \mathbf{F} \cdot d\mathbf{r}. \quad \text{QED}$$