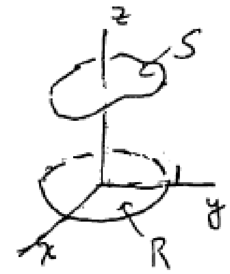
Problems: Stokes' Theorem

1. Let $\mathbf{F} = x^2\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$ and let S be the graph of $z = x^3 + xy^2 + y^4$ over the unit disk. Use Stokes' Theorem to compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where C is the boundary of S.

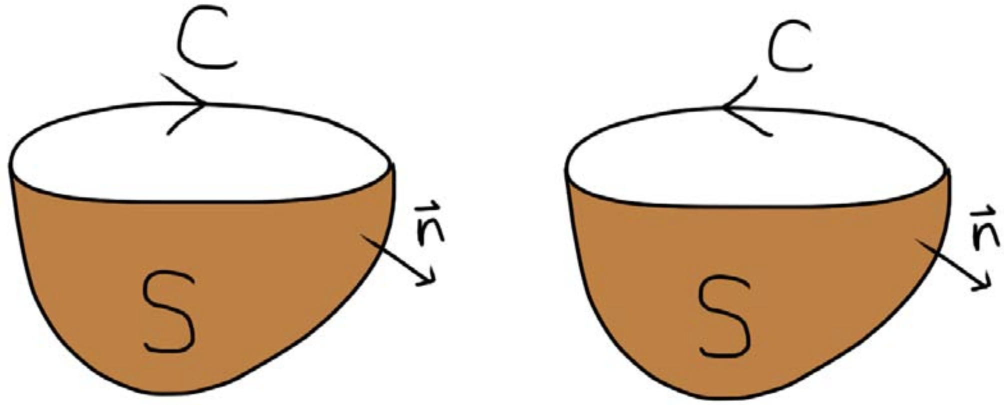


<u>**Answer:**</u> $\text{curl}\mathbf{F} = \langle 0, 0, 1 \rangle$, $\mathbf{n}\, dS = \langle -z_x, -z_y, 1 \rangle\, dx\, dy \Rightarrow \text{curl}\mathbf{F} \cdot \mathbf{n}\, dS = dx\, dy$.

$\Rightarrow \iint_S \text{curl}\mathbf{F} \cdot \mathbf{n}\, dS = \iint_R dx\, dy = \text{area } R = \pi$.

Therefore, by Stokes' Theorem $\boxed{\oint_C \mathbf{F} \cdot d\mathbf{r} = \pi.}$

2. Which of the figures below shows a compatibly oriented surface and curve?



(a) (b)

<u>**Answer:**</u> On surface (a), the curve C is oriented compatibly with the surface S shown. To make this easier to see, add more arrows indicating the orientation of C.