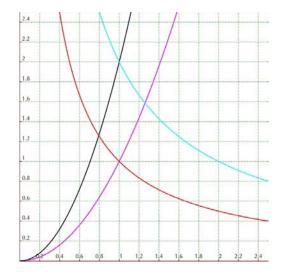# Problems: Change of Variables Example

Use a change of variables to find the area of the region bounded by the curves $y = x^2$, $y = 2x^2$, $y = 1/x$, and $y = 2/x$.

<u>Answer:</u> Draw a picture:

The region is roughly diamond shaped. The top and bottom of the diamond have x coordinate 1, so we could split the area into left and right halves and compute the area using techniques from single variable calculus.

Instead, we rewrite the equations describing the boundary of the region as follows:

$$xy = 1, \quad xy = 2, \quad y/x^2 = 1, \quad y/x^2 = 2.$$

If we let $u = xy$ and $v = y/x^2$, the boundary curves become $u = 1$, $u = 2$, $v = 1$ and $v = 2$. The Jacobian is then:

$$\frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} y & x \\ -\frac{2y}{x^3} & \frac{1}{x^2} \end{vmatrix} = \frac{3y}{x^2} = 3v.$$

Noting that v is positive throughout the region, we get:

$$dx\, dy = \frac{1}{\left|\frac{\partial(u,v)}{\partial(x,y)}\right|} du\, dv = \frac{1}{3v} du\, dv.$$

Finally, we compute:

$$\text{Area} = \iint_R dx\, dy = \int_1^2 \int_1^2 \frac{1}{3v} du\, dv = \frac{1}{3} \ln 2 \approx 0.23.$$

We refer to our original sketch to confirm that this is a plausible result. We could also check our work by computing the area using single variable calculus.

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