Problems: Equation of a Tangent Plane

Find the equation of the plane which is tangent to the surface $x^3 + y^3 + z^3 = 32$ at the point $(1, 2, 3)$.

Answer: : Let $w = x^3 + y^3 + z^3$. We're interested in the surface $w = 32$.
The vector $\boldsymbol{\nabla} w = \langle 3x^2, 3y^2, 3z^2 \rangle$ is normal to this surface, so the normal vector at $(1, 2, 3)$ is $\langle 3, 12, 27 \rangle$.

Applying point normal form for the equation of a plane tells us that:

$$3(x - 1) + 12(y - 2) + 27(z - 3) = 0 \text{ or } 3x + 12\text{y} + 27z = 108$$

is the equation of the tangent plane to $x^3 + y^3 + z^3 = 32$ at $(1, 2, 3)$.

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