Chain rule and total differentials

1. Find the total differential of $w = z\mathrm{e}^{(x+y)}$ at $(0, 0, 1)$.

Answer: The total differential at the point (x_0, y_0, z_0) is

$$dw = w_x(x_0, y_0, z_0)dx + w_y(x_0, y_0, z_0)dy + w_z(x_0, y_0, z_0)dz.$$

In our case,

$$w_x = z\mathrm{e}^{(x+y)}, \quad w_y = z\mathrm{e}^{(x+y)}, \quad w_z = \mathrm{e}^{(x+y)}$$

Substituting in the point $(0, 0, 1)$ we get: $w_x(0, 0, 1) = 1, \quad w_y(0, 0, 1) = 1, \quad w_z(0, 0, 1) = 1.$
Thus,

$$dw = dx + dy + dz.$$

2. Suppose $w = z\mathrm{e}^{(x+y)}$ and $x = t, \quad y = t^2, \quad z = t^3$. Compute $\dfrac{dw}{dt}$ and evaluate it when $t = 2$.

Answer: We use the chain rule:

$$\begin{aligned} \frac{dw}{dt} &= \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} \\ &= (z\mathrm{e}^{(x+y)})(1) + (z\mathrm{e}^{(x+y)})(2t) + (\mathrm{e}^{(x+y)})(3t^2). \end{aligned}$$

At $t = 2$ we have $x = 2, \ y = 4, \ z = 8$. Thus,

$$\left.\frac{dw}{dt}\right|_2 = 8\mathrm{e}^6 + 8\mathrm{e}^6(4) + \mathrm{e}^6(12) = 52\mathrm{e}^6.$$

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