18.02 Problem Set 2

At MIT problem sets are referred to as 'psets'. You will see this term used occasionally within the problems sets.

The 18.02 psets are split into two parts 'part I' and 'part II'. The part I are all taken from the supplementary problems. You will find a link to the supplementary problems and solutions on this website. The intention is that these help the student develop some fluency with concepts and techniques. Students have access to the solutions while they do the problems, so they can check their work or get a little help as they do the problems. After you finish the problems go back and redo the ones for which you needed help from the solutions.

The part II problems are more involved. At MIT the students do not have access to the solutions while they work on the problems. They are encouraged to work together, but they have to write their solutions independently.

Part I (10 points)

At MIT the underlined problems must be done and turned in for grading.
The 'Others' are *some* suggested choices for more practice.

A listing like '§1B : $\underline{2}$, 5$\underline{\text{b}}$, $\underline{10}$' means do the indicated problems from supplementary problems section 1B.

1 Matrices and inverse matrices
§1F: 5$\underline{\text{b}}$, 8$\underline{\text{a}}$; Others: 5a, 9;
§1G: $\underline{3}$, $\underline{4}$ $\underline{5}$;

2 Theorems about square systems. Equations of Planes.
§1H: 3$\underline{\text{abc}}$; Others: 7.
§1E: 1$\underline{\text{cd}}$, $\underline{2}$, $\underline{6}$; Others: 1abe, 3, 5

Part II (17 points)

Problem 1 (5: 1,2,2)
Suppose we know that when the three planes $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ in $\mathbb{R}^3$ intersect in pairs, we get three lines L_1, L_2, and L_3 which are *distinct* and *parallel*.

a) Sketch a picture of this situation.

b) Show that the three normals to $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ all lie in one plane, using a geometric argument.

c) Show that the three normals to $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ all lie in one plane, using an algebraic argument. (Note that the three planes clearly do *not* all intersect at one point.)

Problem 2 (6: 3,1,2)
A manufacturing process mixes three raw materials M_1, M_2, and M_3 to produce three products P_1, P_2, and P_3. The ratios of the amounts of the raw materials (in the order M_1, M_2, M_3) which are used to make up each of the three products are as follows:
For P_1 the ratio is 1: 2: 3; for P_2 the ratio is 1: 3: 5; and for P_3 the ratio is 3: 5: 8.
In a certain production run, 137 units of M_1, 279 units of M_2, and 448 units of M_3 were used. The problem is to determine how many units of each of the products P_1, P_2, and P_3 were produced in that run.

a) Set this problem up in matrix form. Use the letter A for the matrix, and write down the (one-line) formula for the solution in matrix form.

b) Compute the inverse matrix of A and use it to solve for the production vector P.

c) Find a choice for the ratios for the third product (in lowest form), different from the other two ratios, and for which the resulting system has non-unique solutions.

Problem 3 (6: 4,2)
For any plane $\mathcal{P}$ which is not parallel to the x-y plane, define the *steepest direction* on $\mathcal{P}$ to be the direction of any vector which lies in $\mathcal{P}$ and which makes the *largest* (acute) angle with the x-y plane.

a) Let $\mathcal{P}$ be the plane through the origin with normal vector $\mathbf{n}$. Derive a formula, in terms of $\mathbf{n}$, for a vector $\mathbf{w}$ which points in the steepest direction on $\mathcal{P}$

b) Now let $\mathcal{P}$ be the plane through the origin which contains two non-parallel vectors $\mathbf{u}$ and $\mathbf{v}$, where $\mathbf{u}$ and $\mathbf{v}$ do not both lie in the x-y plane. Derive a formula, in terms of $\mathbf{u}$ and $\mathbf{v}$, for a vector $\mathbf{w}$ which points in the steepest direction on $\mathcal{P}$.