Determinants and areas

1. a) Compute $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$.

b) Compute $\begin{vmatrix} 1 & -2 \\ -3 & 4 \end{vmatrix}$.

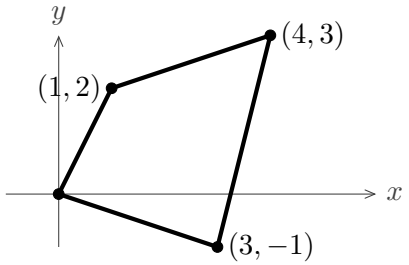
c) Compute $\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$.

Answer: a) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2$.

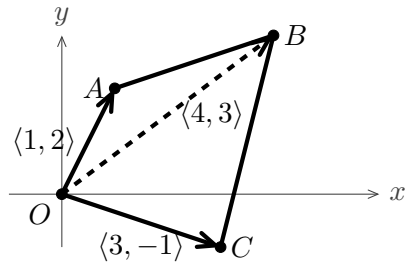
b) $\begin{vmatrix} 1 & -2 \\ -3 & 4 \end{vmatrix} = 1 \cdot 4 - (-2) \cdot (-3) = -2$.

c) $\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 3 \cdot 2 - 4 \cdot 1 = 2$.

2. Find the area of the quadrilateral shown.



Answer:



We break the quadrilateral into two triangles. For convenience, on the figure, we have labeled the vertices $OABC$ and indicated the components of $\overrightarrow{\mathbf{OA}}$, $\overrightarrow{\mathbf{OB}}$ and $\overrightarrow{\mathbf{OC}}$.

Area $\triangle OAB = \frac{1}{2}\left|\det \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}\right| = \frac{1}{2}|-5| = \frac{5}{2}$.

Area $\triangle OBC = \frac{1}{2}\left|\det \begin{pmatrix} 4 & 3 \\ 3 & -1 \end{pmatrix}\right| = \frac{1}{2}|-13| = \frac{13}{2}$.

Thus, area of quadrilateral $OABC = \frac{18}{2} = 9$.

18.02SC Multivariable Calculus
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