THIRD HOMEWORK

Feel free to work with others, but the final write-up should be entirely your own and based on your own understanding.

1. (10 pts) Let L be the line which passes through the point $(a, 0, 0)$ and is parallel to the z-axis. Let D be the region that lies inside the cylinder of radius a centred around the line L and that lies between the planes $z = -1$ and $z = 3$. Describe the region D in cylindrical coordinates.

2. (10 pts) Let D be the region inside the sphere of radius $2a$ centred at the origin and that lies between the planes $x = -a$ and $x = a$. Describe the region D in spherical coordinates.

3. (15 pts) Suppose that $f \colon A \longrightarrow B$ and $g \colon B \longrightarrow C$ are two functions, and let $g \circ f \colon A \longrightarrow C$ be their composition. For each statement below, say whether the statement is true or false. If true, give a reason and if false give a counterexample.

(i) If f and g are surjective, then $g \circ f \colon A \longrightarrow C$ is surjective.

(ii) If $g \circ f \colon A \longrightarrow C$ is surjective, then f is surjective.

(iii) If $g \circ f \colon A \longrightarrow C$ is surjective, then g is surjective.

4. (10 pts) Let $S \subset \mathbb{R}^3$ be the right angled cone, with vertex at the origin and centred around the z-axis, and which lies on or above the xy-plane. Write down a function $f \colon \mathbb{R}^3 \longrightarrow \mathbb{R}$ such that $S = f^{-1}(c)$ is the level set of f at height c.

5. (10 pts) (2.1.34).

6. (5 pts) (2.2.9).

7. (5 pts) (2.2.11).

8. (5pts) (2.2.13).

9. (5 pts) (2.2.15).

10. (5 pts) (2.2.31).

11. (5 pts) (2.2.35).

12. (5 pts) (2.2.42).

Just for fun: What is the volume of the intersection of three cylinders of radius r and height h? Assume that the cylinders are centred around the three coordinate axes and that the central point of each cylinder is the origin.

18.022 Calculus of Several Variables
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