Example: $\int (\ln x)^2\,dx$

To finish learning the method of integration by parts we just need a lot of practice. To this end, we'll do two slightly more complicated examples.

To integrate:

$$\int (\ln x)^2\,dx,$$

assign:

$$\begin{aligned} u &= (\ln x)^2 \qquad & u' &= 2(\ln x)\frac{1}{x} \\ v &= x & v' &= 1. \end{aligned}$$

When we differentiate u we get something simpler, which is a good start. Plugging u and v in to the formula for integration by parts we get:

$$\begin{aligned} \int \underbrace{(\ln x)^2}_{uv'}\,dx &= \underbrace{(\ln x)^2 \cdot x}_{uv} - \int \underbrace{2\ln x \frac{1}{x}}_{u'} \underbrace{x}_{v}\,dx \\ &= x(\ln x)^2 - 2\int \ln x\,dx. \end{aligned}$$

We haven't solved the problem, but we're back to the previous case; we recently computed that $\int \ln x\,dx = x\ln x - x + c$. So we have:

$$\int (\ln x)^2\,dx = x(\ln x)^2 - 2\underbrace{(x\ln x - x)}_{\int \ln x\,dx} + c.$$

As we'll see in the next example, this is typical. Integration by parts frequently involves replacing a "hard" integral by an easier one.

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