Integral of $\tan^3(x)$

Use what you have learned to integrate the function $\tan^3(x)$.

Solution

This is a relatively simple integration; the method described below uses a substitution and the properties $\sec^2 x = 1 + \tan^2 x$ and $\int \tan x \, dx = -\ln|\cos x| + c$.

$$\begin{aligned}
\int \tan^3 x \, dx &= \int \tan x \tan^2 x \, dx \qquad \text{(use an identity to reduce degree)} \\
&= \int \tan x (\sec^2 x - 1) \, dx \\
&= \int \underbrace{\tan x}_{u} \underbrace{\sec^2 x \, dx}_{du} - \int \tan x \, dx \\
&= \int u \, du - (-\ln|\cos x| + c) \\
&= \frac{1}{2}u^2 + \ln|\cos x| + c \\
&= \frac{1}{2}\tan^2 x + \ln|\cos x| + c
\end{aligned}$$

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