Weighted Averages

A *weighted average* is calculated by dividing the weighted total value of a fraction by the total of the weighting function:

$$\frac{\int_a^b f(x)w(x)\,dx}{\int_a^b w(x)\,dx}.$$

Multiplying by $w(x)$ makes some values of $f(x)$ contribute more to the total than other values, depending on the value of x and $w(x)$. Dividing by the integral of $w(x)$ is analogous to dividing by the length or by the number of values.

First we check that this makes sense by confirming that the weighted average of a constant is that same constant:

$$\frac{\int_a^b cw(x)\,dx}{\int_a^b w(x)\,dx} = \frac{c\int_a^b w(x)\,dx}{\int_a^b w(x)\,dx} = c.$$

We see that we were correct to put $\int_a^b w(x)\,dx$ in the denominator.

Now pretend you have a stock which you bought for \$10 one year. Six months later you brought some more for \$20, and then you bought some more for \$30. What's the average price of your stock?

It depends on how many shares you bought. If you bought w_1 shares the first time, w_2 shares the second time and w_3 shares the third time, the total amount that you spent is

$$10w_1 + 20w_2 + 30w_3.$$

The average price per share is the total price divided by the total number of shares:

$$\frac{10w_1 + 20w_2 + 30w_3}{w_1 + w_2 + w_3}$$

This is the discrete analog of the continuous average

$$\frac{\int_a^b f(x)w(x)\,dx}{\int_a^b w(x)\,dx}.$$

The function f is the function describing the price of a share and the weights are the amounts (relative importance) of the different purchases.

Question: You can't factor out the $f(x)$, can you?

Answer: When we found the weighted average of a constant, we factored out c. In

$$\frac{\int_a^b f(x)w(x)\,dx}{\int_a^b w(x)\,dx}$$

we cannot factor out $f(x)$. If the weighted average is interesting you have to do two different integrals to calculate it. It's only when $f(x)$ is constant that you can factor it out (in which case, the calculation is not very interesting at all).

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