

**PROFESSOR:** Hi. Welcome back to recitation. In lecture you've been doing definite integration for the last couple of lectures. And you've just started with the fundamental theorem of calculus. So, that gives us this whole powerful tool to compute a bunch of integrals. So, with that in mind, I've got a couple questions for you.

So compute the definite integrals. The integral from  $\pi/6$  to  $\pi/3$  of  $\tan x \, dx$ . And the integral of  $-\pi/3$  to  $\pi/3$  of  $\tan x \, dx$ . So why don't you pause the video, take a couple minutes to work on those, come back and we'll work on them together.

All right. Welcome back. So, as we were saying we have a couple of definite integrals here. So one tool for definite integrals is Riemann sums. So, in principle, we could write down a Riemann sum for, say, this first integral here. And then try and compute it by trying to compute the sum and taking a limit of it. But I think it's pretty clear that, for a function like  $\tan x$ , that's going to be really, really hard to do.

So we want to not use Riemann sums here. And the way we can not use Riemann sums is using this great tool that we have, which is the fundamental theorem of calculus. So the fundamental theorem of calculus says, when you have a definite integral, instead of computing Riemann sums, what you can do is compute an antiderivative and use the antiderivative to find out what the integral is. Just compare two values of the antiderivative and that'll give you the value of the definite integral.

So, in particular, let's do this first integral first. So we have the integral from  $\pi/6$  to  $\pi/3$  of  $\tan x \, dx$ . Well, so, we know what the antiderivative of  $\tan x$  is. We did that in an earlier recitation. So that's  $\ln$  of  $\cosine x$ . Or  $\ln$  of the absolute value of  $\cosine x$ . So, the fundamental theorem of calculus says that the value of this definite integral, in order to compute it, we just take the difference of that antiderivative at  $\pi/3$  and at  $\pi/6$ .

So, by the fundamental theorem of calculus this is equal to  $\ln$  of the absolute value of  $\cosine x$  for  $x$  between  $\pi/6$  and  $\pi/3$ . So, OK, so this, you know, this is this notation that Professor Jerison introduced. So what does this actually mean? It's just a shorthand for  $\ln$  of the absolute value of  $\cosine$  of  $\pi/3$  minus  $\ln$  of the absolute value of  $\cosine$  of  $\pi/6$ .

Now, we could leave our answer like this but we can also manipulate it and put it in a little nicer form. So,  $\cosine$  of  $\pi/3$  is-- oh, I'm sorry I've made a mistake. So it's not  $\ln$  of absolute

value  $\cos x$ . It's minus  $\ln$  of absolute value of  $\cos x$ . Sorry about that. So this is minus  $\ln \cos \frac{\pi}{3}$  plus-- OK, so minus and minus there is plus. All right. Sorry about that. OK, so, good. All right.

And we can pick up where I was in the middle of my sentence and OK, so we can just evaluate this by just evaluating. So  $\cos \frac{\pi}{3}$ , that's going to be  $\frac{1}{2}$ . So this is minus  $\ln$  and absolute value of  $\frac{1}{2}$  is just a  $\frac{1}{2}$ . Plus  $\cos \frac{\pi}{6}$  is square root of 3 over 2. So plus  $\ln$  square root of 3 over 2. And now you can use your logarithm rules to combine those into a single expression.

So this is equal to  $\ln$  of square root of 3. Or you could even use one more logarithm rule if you wanted. So this is equal to  $\frac{1}{2} \ln$  of 3. So all of those are, you know, equivalent expressions for the same number, which is the value of this definite integral.

OK, so, for the second integral now, we can do exactly the same thing if we wanted to. And it'll be a very similar process. You'll write down this antiderivative here, you'll take the difference in values. We can also do something a little bit clever for this one. Which is, definite integrals have some geometric interpretation, right? And if you think about the function  $\tan x$ -- so we could put up a little graph here-- so in between minus  $\frac{\pi}{3}$  and  $\frac{\pi}{3}$ ,  $\tan x$  maybe looks something like this. Yeah, you know, give or take.

OK, so what's important about this? Well  $\tan x$  is an odd function. And this interval from-- this is minus  $\frac{\pi}{3}$  to  $\frac{\pi}{3}$ , and this is the curve  $y = \tan x$ . So this is an odd function. So the integral in question is this positive area minus this negative area. But since it's an odd function, it's symmetric. So those two cancel out perfectly. So for the second one, you could do the same process that we did. But you can also use a little bit of geometric reasoning to realize that this second one is just equal to 0.

In either case, doing either of these integrals with Riemann sums would be really atrocious. So, in both cases, what we're really happy about is that we have these other tools that allow us to evaluate definite integrals without going back to the Riemann definition of the integral. So, I'm going to stop there.